

NAME:

Math 103 Exam 2 retake

12 December 2008

100 points

Instructions:

1. This exam has 6 pages (including this one), which contain 8 problems and one bonus problem. Please check that you have all of the pages.
 2. Answer all of the following questions clearly and completely. Justify all of your answers.
 3. You may not use a book or any notes for this exam.
 4. Give your answer to each problem completely and clearly in the space provided. You may use the back of the exam pages for scratch work; however, if you want this work to be considered, make note of it in the space provided for the problem.
 5. Erase or cross out work you do not wish to be graded.
 6. Credit, partial or full, will be given only if sufficient steps leading to the answers are shown.
 7. You have 50 minutes to complete this exam.
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Problem 1. (17 points) Let $f(x) = 4x^3 - 12x^2 - 7x + 15$.

(a) (3 points) What is the maximum possible number of zeros f may have? Why?

(b) (8 points) Find all possible rational zeros of f . (You do *not* have to test them to see if they're actually zeros.)

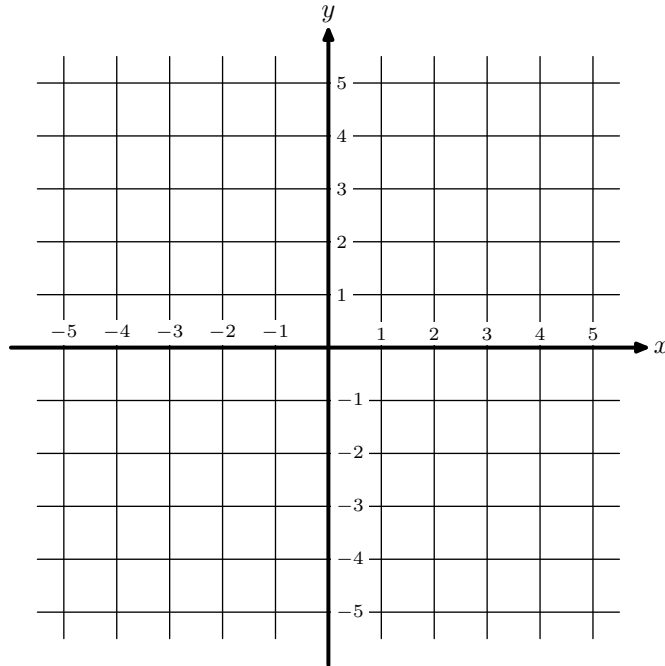
(c) (6 points) Without graphing, prove that f must have a zero somewhere between $x = 3$ and $x = 4$.

Problem 2. (10 points) Let $f(x) = 9x^5 - 3x + 2$ and $g(x) = 4 - x$. Find $(f \circ g)(x)$. You do not have to simplify.

Problem 3. (10 points) Let f be the piecewise-defined function given by

$$f(x) = \begin{cases} -x^2 + 4, & \text{if } x < 0; \\ \sqrt{x}, & \text{if } 0 \leq x < 4. \end{cases}$$

Graph $y = f(x)$.

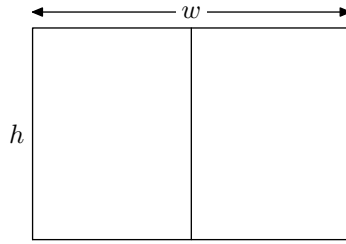


Problem 4. (10 points) Give a polynomial function $h(x)$ that satisfies *all* of the following conditions:

- The zeros of h are $x = 1$ and $x = -3$ (and no other real values of x).
- The graph of h crosses the x -axis at $x = 1$ but just touches the x -axis at $x = -3$.
- The degree of h is at least 5.

You do not need to simplify your expression for h .

Problem 5. (16 points) You have a total of 1800 feet of fencing. You want to build a rectangular corral with two halves, as shown in the picture below.



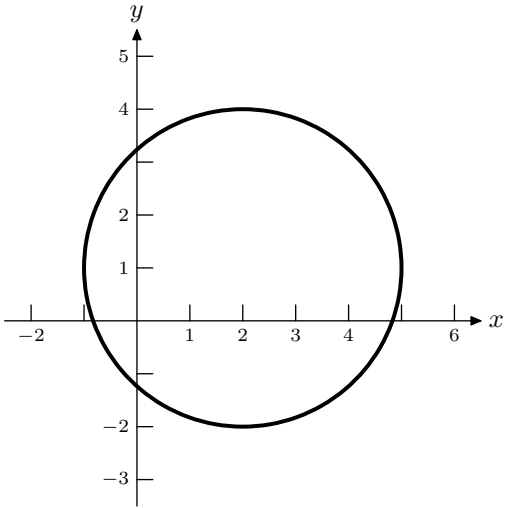
(a) (5 points) Write an equation that relates the variables w and h . [Hint: Use the fact that the total length of the fencing should be 1800 feet.]

(b) (5 points) Express the total area A of the corral (both halves together) as a function of w . [Hint: Your answer from part (a) should help you.]

(c) (2 points) You should have a quadratic function as your answer to part (b). Its graph is a parabola. Does this parabola open up or open down? Why?

(d) (4 points) What dimensions w and h will give you the largest enclosed area?

Problem 7. (5 points) Is the following graph the graph of a function? If so, what function? If not, why not?



Problem 8. (12 points) Solve the following inequality for x .

$$(x - 5)^2(x + 3) \leq 0$$

Bonus problem (+3 points). Explain why every polynomial of odd degree must have at least one real zero. [Hint: Think about the end behavior of the graph.]