NAME:

MATH 103 Exam 2, version (a)

3 October 2008

100 points

Instructions:

1. This exam has 6 pages (including this one), which contain 7 problems and one bonus problem. Please check that you have all of the pages.
2. Answer all of the following questions clearly and completely. Justify all of your answers.
3. You may not use a book or any notes for this exam.
4. Give your answer to each problem completely and clearly in the space provided. You may use the back of the exam pages for scratch work; however, if you want this work to be considered, make note of it in the space provided for the problem.
5. Erase or cross out work you do not wish to be graded.
6. Credit, partial or full, will be given only if sufficient steps leading to the answers are shown.
7. You have 50 minutes to complete this exam.

Problem 1. (21 points) Let $f(x) = 5x^5 - 32x^4 + 22x^3 + 96x^2 - 16$.

(a) (3 points) What is the maximum possible number of zeros f may have? Why?

(b) (8 points) Find all possible rational zeros of f. (You do *not* have to test them to see if they're actually zeros.)

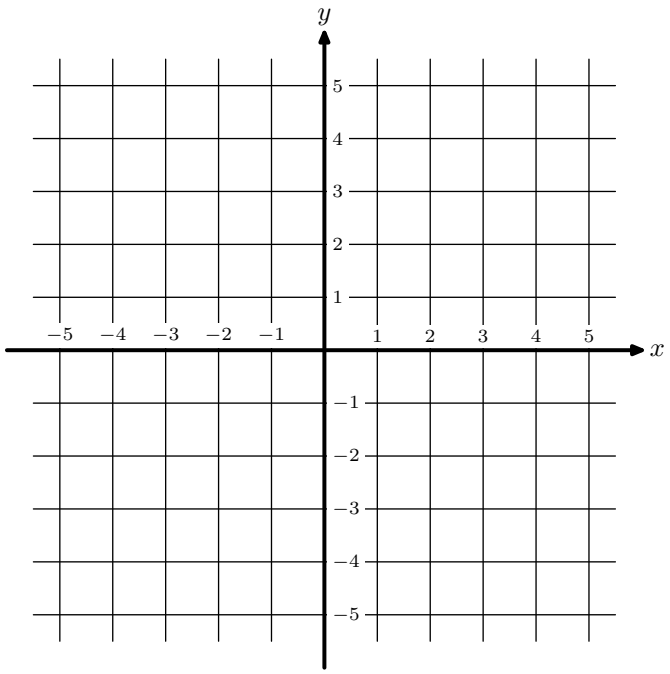
(c) (6 points) Without graphing, prove that f must have a zero somewhere between $x = 4$ and $x = 5$.

(d) (4 points) Explain why the zero described in part (c) must be at an *irrational* (not rational) value of x.

Problem 2. (10 points) Let f be the piecewise-defined function given by

$$f(x) = \begin{cases} 1, & \text{if } x < -1; \\ -x^2 + 2, & \text{if } -1 \leq x < 2; \\ 2x - 7, & \text{if } 2 < x < 4. \end{cases}$$

Graph $y = f(x)$.



Problem 3. (8 points) Give a polynomial function $h(x)$ that satisfies *both* of the following conditions:

- The zeros of h are $x = 3$ and $x = -7$ (and no other real values of x).
- The graph of h crosses the x-axis at $x = 3$ but just touches the x-axis at $x = -7$.

You do not need to simplify your expression for h.

Problem 4. (12 points) Solve the following inequality for x.

$$(x-2)(x+3)^2 \geq 0$$

Problem 5. (14 points) Let $g(x) = 2x^2 - 5x$.

(a) (7 points) Find the average rate of change of g from $x = -1$ to $x = 3$.

(b) (7 points) Find the equation of the secant line containing the points $\big(-1, g(-1)\big)$ and $\big(3, g(3)\big)$.

Problem 6. (25 points) Here we go. This is the problem everyone has been looking forward to. Let

$$R(x) = \frac{(x+1)(2x-1)}{(x-2)(x+1)}.$$

(a) (3 points) What is the domain of R?

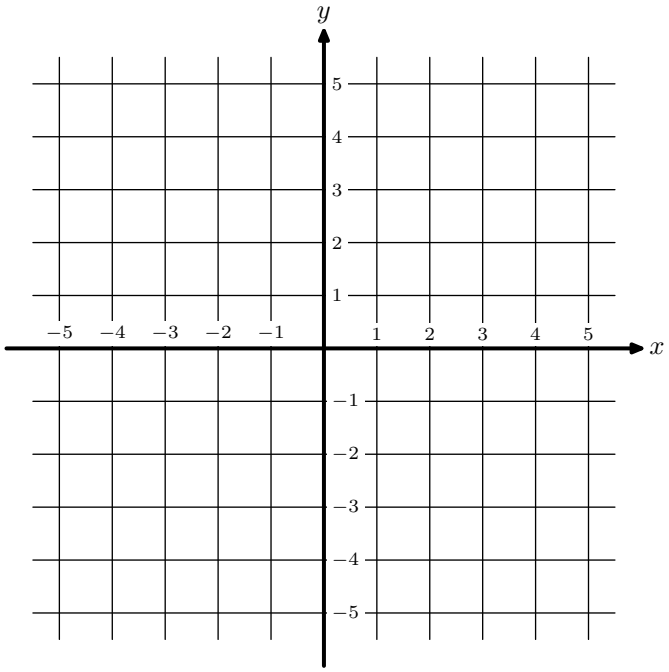
(b) (3 points) If 0 is in the domain of R, what is the y-intercept of R? Plot this point on the graph below.

(c) (3 points) Write R in lowest terms.

(d) (3 points) Find the x-intercept(s) of R. Plot the x-intercept(s) on the graph below.

(e) (4 points) Find any vertical asymptotes of R, and sketch them on the graph below.

(f) (4 points) Find the horizontal or oblique asymptote of R, and sketch it on the graph.



(g) (5 points) Complete the graph of R. You will probably have to plot at least one more point in order to understand how the function behaves. [Don't forget about your answer to part (a).]

Problem 7. (10 points) Let $f(x) = 2x^3 + x^2 - 4$ and $g(x) = 5x - 1$. Find $(f \circ g)(x)$. You do not have to simplify.

Bonus problem. (+3 points) Explain why every polynomial of odd degree must have at least one real zero. [Hint: Think about the end behavior of the graph.]