Corollary 2.2 and Example 2.3

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Corollary 2.2

Let f be a polynomial of $Int(D)$ such that

(i) f is irreducible in $K[X]$

(ii) for every maximal ideal M of D, there exists $\alpha \in D$ such that $f(\alpha) \notin M$.

Then f is irreducible in $Int(D)$.

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Recall Proposition 2.1.

Let f be a polynomial of $Int(D)$ which is irreducible in $K[x]$. Then the following assertions are equivalent:

(i) f is irreducible in $Int(D)$.

(ii) For each $a \in D$, $(f/a) \in Int(D)$ if and only if a is a unit of D.

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Therefore, a is a unit.
Also notice that if a is a unit then $(f/a) \in Int(D)$.

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Thus, $a \notin M$.
Therefore, a is a unit.
Also notice that if a is a unit then $(f/a) \in Int(D)$.
By applying Proposition 2.1, f is irreducible in $Int(D)$. □

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- for every maximal ideal M of D, there exists $\alpha \in D$ such that $f(\alpha) \notin M$.

$\therefore X - a$ is irreducible in $Int(D)$.