1. Prove that every PID is a Dedekind Domain.

2. Let $R$ be a Dedekind Domain. Prove that $R$ is a PID if and only if $R$ is a UFD.

3. Let $I(R)$ denote the set of nonzero fractional ideals of a Dedekind domain $R$ and let $P(R)$ denote the set of nonzero principal fractional ideals of $R$.
   (a) Prove that $I(R)$ is a multiplicative abelian group.
   (b) Prove that $P(R)$ is a subgroup of $I(R)$.
   (c) The ideal class group $C(R)$ is the quotient group $I(R)/P(R)$. Prove that $C(R)$ is trivial if and only if $R$ is a PID.
   (d) Note that, in fact, the ideal class group of a Dedekind domain $R$ gives a measure of how far away $R$ is from being a UFD. Indeed, $R$ is a UFD if and only if $C(R)$ is trivial and $R$ is a HFD if and only if $|C(R)| \leq 2$. (You do NOT need to prove this result.)

4. Let $F$ be any field and $f$ be an irreducible polynomial in $F[x]$. Define $F[x]_f = \{ \frac{g}{h} \mid g, h \in F[x], h \neq 0, f \nmid h \}$. That is,
   $F[x]_f = \{ \text{all rational functions in } F(x) \text{ whose denominator is not divisible by } f \}$.
   Check that $\nu$ is discrete valuation on $F(x)$ and that the corresponding valuation ring is $F[x]_f$.

5. Let $R$ be a DVR. Prove that $R$ is a Euclidian Domain. (Hint: Start by proving that the discrete valuation, restricted to $R$; $\nu : R \rightarrow \mathbb{N}_0$ is a norm.)