

If the residue field  $k$  is not algebraically closed, the normal forms for the  $(A_1)$ -singularities in Theorem 1.1 and in (3) of Theorem 3.1 must be replaced by the following:

$$(\dagger) \quad k[[x_0, \dots, x_d]] / (g + v_1 x_2^2 + v_1 v_2 x_3^2 + \dots + v_1 v_2 v_3 \cdot \dots \cdot v_{d-1} x_d^2),$$

where each  $v_i$  is a unit of  $k[[x_0, \dots, x_i]]$  and  $g \in k[[x_0, x_1]]$ . By adjusting  $g$  and multiplying the defining equation by  $v_1^{-1}$ , we could eliminate the unit  $v_1$ . Therefore the forms given in Theorems 1.1 and 3.1 are valid for one-dimensional rings.