1. Find parametric equations for the tangent line to the parametric curve 
\[ x(t) = (t \cosh t, t \sinh t, t) \]
at the point where \( t = 0 \).

**Solution:** We need \( x'(0) \) (which is a vector parallel to the tangent line) and \( x(0) = (0, 0, 0) \) (which is a point on the line).

\[ x'(t) = \frac{dx}{dt} = (\cosh t + t \sinh t, \sinh t + t \cosh t, 1) \]

and so
\[ x'(0) = (1 + 0, 0 + 0, 1) = (1, 0, 1). \]
The line has parametric equations
\[
\begin{cases} 
  x &= 0 + 1t \\
  y &= 0 + 0t \\
  z &= 0 + 1t 
\end{cases}
\]
or
\[
\begin{cases} 
  x &= t \\
  y &= 0 \\
  z &= t 
\end{cases}
\]

2. Find the partial derivatives with respect to \( x \) and \( y \) evaluated at \((x_0, y_0) = (2, -2)\) for
\[ f(x, y) = \frac{x \cos(\pi y)}{x^2 + y^2} \]

**Solution:** We could do this
\[
\begin{align*}
  f(x, y_0) &= f(x, -2) \\
              &= \frac{x \cos(-2\pi)}{x^2 + (-2)^2} \\
              &= \frac{x}{x^2 + 4} \\
  \frac{\partial f}{\partial x} \bigg|_{(x, -2)} &= \frac{d}{dx} f(x, y_0) = \frac{d}{dx} \frac{x}{x^2 + 4} \\
                                      &= \frac{(x^2 + 4) - x(2x)}{(x^2 + 4)^2} \\
                                      &= \frac{4 - x^2}{(x^2 + 4)^2} \\
  \frac{\partial f}{\partial x} \bigg|_{(2, -2)} &= 0
\end{align*}
\]
and this

\[ f(x_0, y) = f(2, y) = \frac{2 \cos(\pi y)}{4 + y^2} \]

\[ \frac{\partial f}{\partial y} \bigg|_{(2,y)} = \frac{d}{dy} f(x_0, y) = \frac{d}{dy} \frac{2 \cos(\pi y)}{4 + y^2} = -2\pi \sin(\pi y)(4 + y^2) - 2 \cos(\pi y)(2y) \]

\[ \frac{\partial f}{\partial x} \bigg|_{(2,-2)} = 0 + \frac{8}{8^2} = 1 \]


But this is how we would usually do it:

\[ \frac{\partial f}{\partial x} = \frac{\cos(\pi y)(x^2 + y^2) - x \cos(\pi y)(2x)}{(x^2 + y^2)^2} \]

\[ = \frac{(x^2 + y^2 - 2x^2) \cos(\pi y)}{(x^2 + y^2)^2} \]

\[ = \frac{(y^2 - x^2) \cos(\pi y)}{(x^2 + y^2)^2} \]

\[ \frac{\partial f}{\partial y} = \frac{-\pi x \sin(\pi y)(x^2 + y^2) - x \cos(\pi y)(2y)}{(x^2 + y^2)^2} \]

\[ = \frac{-\pi x(x^2 + y^2) \sin(\pi y) - 2xy \cos(\pi y)}{(x^2 + y^2)^2} \]

\[ \frac{\partial f}{\partial x} \bigg|_{(2,-2)} = 0 \]

\[ \frac{\partial f}{\partial y} \bigg|_{(2,-2)} = \frac{0 + 8}{8^2} = \frac{1}{8} \]

3. Find the partials \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) evaluated at \((x_0, y_0)\) (which means \( \frac{\partial f}{\partial x} \bigg|_{(x_0,y_0)} = f_x(x_0, y_0) \) and \( \frac{\partial f}{\partial y} \bigg|_{(x_0,y_0)} = f_y(x_0, y_0) \)) for the following cases.

(a) \( f(x, y) = e^{(x+2y^3)}, (x_0, y_0) = (-1, 1) \)
Solution:

\[
\frac{\partial f}{\partial x} = e^{(x+2y-3)} \frac{\partial}{\partial x} (x + 2y - 3) = e^{(x+2y-3)}
\]

\[
\frac{\partial f}{\partial y} = e^{(x+2y-3)} \frac{\partial}{\partial y} (x + 2y - 3) = 2e^{(x+2y-3)}
\]

\[
\frac{\partial f}{\partial x} \bigg|_{(-1,1)} = e^{-2}
\]

\[
\frac{\partial f}{\partial y} \bigg|_{(-1,1)} = 2e^{-2}
\]

(b) \( f(x, y) = \sqrt{x^2 + y^2}, \ (x_0, y_0) = (-3, 4) \)

Solution:

\[
\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} (2x) = \frac{x}{\sqrt{x^2 + y^2}}
\]

\[
\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} (2y) = \frac{y}{\sqrt{x^2 + y^2}}
\]

\[
\frac{\partial f}{\partial x} \bigg|_{(-3,4)} = -\frac{3}{5}
\]

\[
\frac{\partial f}{\partial y} \bigg|_{(-3,4)} = \frac{4}{5}
\]