Assignment 3 - Due Friday, March 15

1. Determine if the systems below could be interpreted as models for predatory-prey, competing, or cooperating species. Motivate your answer.
   (a) \( x' = x(1 - x + y), \quad y' = y(4 - 3y - x). \)
   (b) \( x' = x(1 - x - y), \quad y' = y(4 - 3y - x). \)
   (c) \( x' = x(1 - x + y), \quad y' = y(4 - 3y + x). \)

2. Solve the following initial value problem:
   \[
   \mathbf{X}'(t) = \begin{bmatrix} 5 & -5 \\ 1 & 1 \end{bmatrix} \mathbf{X}(t), \quad \mathbf{X}(0) = \begin{bmatrix} -5 \\ 2 \end{bmatrix}.
   \]

3. Compute the matrix exponential \( e^{tA} \) in the case when \( A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \).

4. For the differential system \( \mathbf{x}'(t) = A\mathbf{x}(t) \):
   (a) Draw the nullclines and perform a nullcline analysis;
   (b) Find the general solution;
   (c) Discuss the stability of the origin based on parts (a) and (b);
   (d) Draw some trajectories to illustrate what type of a critical point the origin is.

   where
   (a) \( A = \begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix} \)
   (b) \( A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix} \)
   (c) \( A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} \)

5. Solve the following initial value problems:
   (a) \( x' = 5x - y, \quad y' = 3x + y \) with \( x(0) = 2, \quad y(0) = -1. \)
   (b) \( x' = x - 5y, \quad y' = x - 3y \) with \( x(0) = 1, \quad y(0) = 1. \)
   (c) \( x' = 3x + 9y, \quad y' = -x - 3y \) with \( x(0) = 2, \quad y(0) = 4. \)

6. Two interconnected tanks contain a solution of salt and water. Water containing 1lb salt per gallon flows into Tank 1 at a rate of 5 gal/min and into Tank 2 at a rate of 7 gal/min. Some of the mixture in Tank 1 flows out carrying every minute 2/7 of the amount of salt present in Tank 1 at time \( t \). Half of this amount flows into Tank 2 and the other half leaves the system. Some of the mixture in Tank 2 flows out carrying 1/4 of the amount of salt present in Tank 2 at time \( t \). Denote by \( x(t) \) and \( y(t) \) the amounts of salt at time \( t \) in Tank 1, respectively, Tank 2. Write a differential system of equations that model the flow process.