398 Math in the City
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Homework 8

Due April 11

1. (6 points) Consider the nonlinear system

$$\begin{cases} \dfrac{dx}{dt} = \varepsilon x - y \\ \dfrac{dy}{dt} = x + \varepsilon y. \end{cases},$$

Show that the critical point (0,0) is (a) stable spiral point if $\varepsilon < 0$; (b) center if $\varepsilon = 0$; (c) an unstable spiral point if $\varepsilon > 0$. Thus small perturbations of the system $x' = -y, \quad y' = x$ can change both the type and the stability of the critical point. Use Maple to illustrate the loss of stability that occurs at $\varepsilon = 0$ as the parameter increases from $\varepsilon < 0$ to $\varepsilon > 0$.

2. (5 points) The differential equation

$$\frac{dx}{dt} = \frac{1}{10}x(10 - x) + s$$

models a (logistic) population with stocking at rate s.Determine the dependence of the number of critical points c on the parameter s and then construct the corresponding bifurcation diagram in the sc plane. For each case, use Maple to plot the slope field of solutions.

3. (4 points) Consider the two differential equations

$$\frac{dx}{dt} = (x - a)(x - b)(x - c)$$

and

$$\frac{dx}{dt} = (a - x)(b - x)(c - x),$$

each having the critical points a, b and c; suppose that $a < b < c$. For one of these equations, only the critical point b is stable; for the other equation, b is the only unstable critical point. Use phase line analysis for the two equations to determine which is which. Without attempting to solve either equation explicitely, make rough sketches of typical solutions for each. You should see two funnels and a spout in one case, two spouts and a funnel in the other.