

## Homework 7

Due April 3

1. (5 points) Consider the nonlinear system of DEs:

$$\begin{cases} \frac{dx}{dt} = x\sqrt{x+y+1} \\ \frac{dy}{dt} = x(x+y+1). \end{cases}, \quad \begin{cases} \frac{dx}{dt} = -y^2 - \ln x \\ \frac{dy}{dt} = x^3 + \frac{y}{x} \end{cases}$$

- (a) Find the equations of the trajectories for the above systems.  
(b) For each system, compute the slopes of the minitangents in the phase portrait at  $(1, 0)$  and  $(0, -1)$ . Are they well defined? If not, do you think that the solutions  $(x(t), y(t))$  do not exist at those point?
2. (5 points) A body with mass  $m = 0.5$  kg is attached to the end of a spring that is stretched 2 m by a force of 100 N. It is set in motion with initial position  $x_0 = 1$  m and initial velocity  $v_0 = -5$  m/s. (Note that these initial conditions indicate that the body is displaced to the right and is moving to the left at time  $t = 0$ .) Find the position function of the body as well as the amplitude, frequency, and period of oscillation.
3. (5 points) Write about your experience with the project you worked on this semester. Make sure you touch on the following points: things you learnt, work that you enjoyed, the mathematical (and nonmathematical) difficulties that you encountered along the way, long term benefits you think you will have after completing the work. Write out the mathematical details of the work that you found most interesting. (Again, no collaboration, type and print out this part of the homework).