

Homework 6 Due March 22

1. (5 points) The following linear model is used to predict the early development of epidemics in underdeveloped countries:

$$\begin{cases} I' = \frac{3}{125 - 5t}(3I + 2U - Q) \\ Q' = \frac{1}{5}(I - Q) \\ U' = -(U + I), \end{cases}$$

where I , Q , and U represent the number of infected, quarantined, and uninfected individuals. The model gives a good prediction for the first 5 days after the outbreak. Use the improved Euler's Method with step size $h = 0.1$ to predict the values of I , Q , and U at $t = 5$ when the initial data is given by $I_0 = 41$, $Q_0 = 10$, $U_0 = 1018$.

(a) Change the initial data in the way described below and notice what happens with the final values for the three variables I , Q , and U . First increase the initial value of I and keep the other values constant. In a second step, increase the value of Q and keep I and U constant. Finally, compute the change in the final values when you increase the initial population of uninfected people.

(b) For the given initial value problem notice that the number of infected people tends to stabilize after some time. Use this observation to predict the number of days a health organization would have to secure the quarantine in order to contain the disease.

(c) What happens if the quarantine process is less successful (e.g. $Q' = \frac{1}{10}(I - Q)$, so the rate of change is smaller)? Would a health organization still be able to contain the disease?

2. (5 points) For each of the following nonlinear systems:

$$\begin{cases} x' = x^2 - y^2 \\ y' = 2x - 3y \end{cases}, \quad \begin{cases} r' = r - 2rp \\ p' = \frac{1}{2}r^2 - p \end{cases}$$

(a) Find the critical points and the nullclines (here, they will be curves, instead of straight lines, like in the case of a linear system). Linearize the system. Can you draw any conclusions regarding the stability of the critical points of the nonlinear system based on the Hartman-Grobman theorem?

(b) Use Maple to draw the phase plane for the nonlinear system and for its linearized version. Draw some trajectories in each case.

3. (5 points) In 15-20 lines describe the work that you have done on the project so far, the observations and the conclusions that can be drawn out of it. Again, you may not consult your teammates on this task. Write down everything that you noticed, even if you are

uncertain of the value of your observations. By redoing a mental analysis of the work you performed so far, most likely you will find things of interest that escaped your analysis before. Type the write-up of this exercise, print it and attach it to the homework (of course, do not forget to save the file!).