398 Math in the City
Instructor: Petronela Radu
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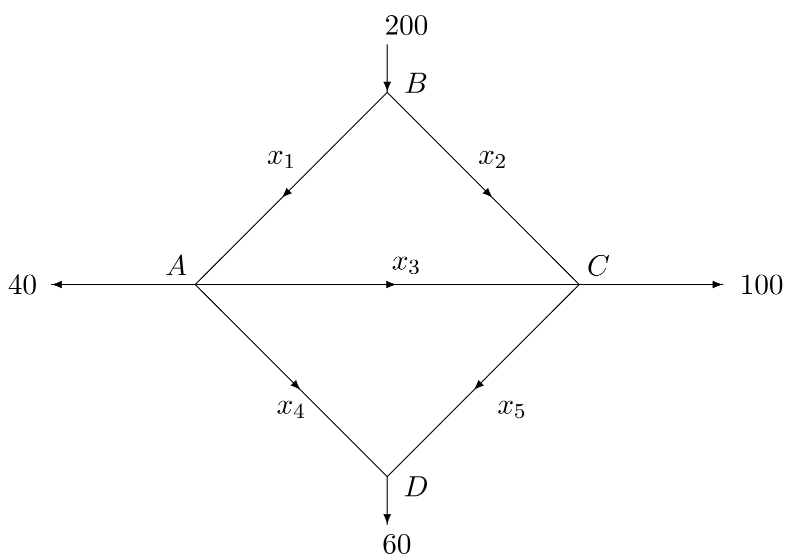
Homework 1

Due January 30

1. (30 points) What condition must be placed on a, b, and c so that the following system in unknowns x, y, and z has a solution?

$$\begin{cases} x + 2y - 3z = a \\ 2x + 6y - 11z = b \\ x - 2y + 7z = c \end{cases}$$

2. (a) (30 points) Find the general traffic pattern in the freeway network shown in the figure. (Flow rates are in cars/minute.)
   (b) Describe the general traffic pattern when the road whose flow is x_4 is closed.
   (c) When $x_4 = 0$, what is the minimum value of x_1?



3. (10 points) Alka-Seltzer contains sodium bicarbonate ($NaHCO_3$) and citric acid ($H_3C_6H_5O_7$). When a tablet is dissolved in water, the following reaction produces sodium citrate, water, and carbon dioxide (gas):

   $NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$.

   Balance the chemical equation by using the vector equation approach discussed in class.

4. (10 points) Show that if λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2.

5. (40 points) Consider the matrix

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) Find the eigenvalues of the matrix A.

(b) Find a basis for each of the eigenspaces of A.

(c) Write the characteristic equation for A and explain why A is diagonalizable.

(d) Diagonalize A.

6. (15 points) Find the matrix of the linear transformation that deforms the square $[0,2] \times [0,2]$ into the parallelogram with vertices at the points (0,0), (2,2), (2008,2), (2006,0). Find the area of the parallelogram by using the theorem about the determinant of the matrix of the linear transformation.

7. (15 points) Show that 2, $2-t$, and t^2 form a basis for $\mathbb{P}_2$ (the set of polynomials of degree at most 2).