

Sample Problems for Test 1

1. For what value(s) of λ does the DE:

$$\frac{dy}{dx} = xy$$

have a solution of the form $y(x) = e^{-\lambda x^2}$?

2. Find the explicit solution of the following initial value problem and state precisely the interval of existence:

$$y' = -\frac{t}{t^2 - 4}y, \quad y(1) = 1.$$

3. Sketch the set of points in the ty plane where the slope field of the differential equation $y' + 2x = 3y$ has the value 3.

4. Find a function $y = y(t)$ that solves the IVP

$$y' = \sqrt{t}e^t, \quad y(1) = 2.$$

5. With two different methods solve the IVP

$$y' = -my + h, \quad y(0) = y_0.$$

6. Use the substitution $v = a + b^2t + y$ to solve

$$y' = (a + b^2t + y)^2, \quad y(0) = 1.$$

7. Consider a population P whose growth is modeled by a logistic DE with carrying capacity 100. If at time $t = 0$ the population has a rate of growth of 120 individuals per week and the initial population is 40,

- Write an IVP for this system;
- How would the DE from part (a) change if we assume that the population is harvested at a rate of 50 individuals per week?
- Perform a phase line analysis for the model obtained in part (b), and then solve the equation with initial condition $P(0) = 5$.

8. Solve the following DEs:

(a) $(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$ (exact)

(b) $\frac{dy}{dx} = \frac{y - 4x}{x - y}$ (homogeneous)

(c) $y^3y'' = 1$ (reducible second order).

9. (15 points) Use Euler's method with step $h = .25$ to find an approximation of the solution to the IVP

$$y' = \frac{y^2 + t}{4t + 1}, \quad y(1) = 2,$$

at time $t = 2$.

10. (15 points) Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. The coffee has a temperature of 80°C when freshly poured, and 1 min later has cooled to 70°C , when the temperature outdoors is 0°C .
- Write an initial value problem for this model.
 - Determine the time when the coffee reaches a temperature of 40°C .
(Note: Begin the solution by stating your notation).
11. Problem 34 page 54.
12. Problem 19 page 107.