

Project 2 - Due December 1

You may work on this project individually, in groups of two, three, or four. If this is a group project, write how each of the group members contributed to the final work.

The project covers concepts such as modelling, quantitative and qualitative analysis of a system of differential equations. Read your notes and the textbook material from the sections covered in chapters 5 and 6.

Provide clear solutions for each of the problems and show how you obtained your results. Your solution should be written using complete and correct sentences, your notation should be consistent.

A Waste Treatment Process that Utilizes Bacteria

Let $w(t)$ be the amount of waste to be treated in a vessel, and let $x(t)$ be the population of bacteria in the vessel at time t . Assume that the waste is consumed at a rate proportional with the population of bacteria (take the proportionality constant $k > 0$). Let c be a conversion factor for bacteria growth from waste consumption, and let m be the relative death rate of the bacteria.

Task 1. Construct the differential equations for the waste and the bacteria. Explain why the above assumptions are reasonable, motivate the presence (and the absence!) of the terms in x and w from your DEs. What type of interacting populations are the waste and the bacteria in this model: predator-prey, cooperating, or competing? Justify your answer.

Task 2. Solve the system of differential equations for all possible cases of the parameters $k, c, m \geq 0$.

Task 3. Discuss the stability of the system in all of the above cases.

Task 4. In each of the cases found in Task 2, choose some values for c, k and m to plot the slope field for the system (use a computer system like *Mathematica* or *Maple*). Draw in each case some trajectories in the phase plane and illustrate how a different choice of solutions changes the outcome for the two interacting populations.

Extra Credit. For **one** extra credit point provide a different model (possibly nonlinear) that could model waste treatment and explain why your model could provide more accurate predictions about the long time behavior of the waste and bacteria.