

Homework 4

Due November 1

1. (40 points) Use *pdsolve* in Maple to find the general solution to the following partial differential equations:

- (a) $u_t - 4u_x = 0$
- (b) $u_t - 4u_x = u^2$
- (c) $u_t - 4u_{xx} = 0$
- (d) $u_t - 4xtu_x = u$

In each case choose an initial condition (i.e. let $u(0)$ be a given function of your choice) and find the corresponding particular solution. Plot the particular solution as a surface. Print out your Maple program and also show your work for finding the particular solutions.

2. (50 points) Consider the predator-prey system of owls and rats given as an example in class of a discrete dynamical system (see handout)

$$\begin{cases} O_{k+1} = (.5)O_k + (.4)R_k \\ R_{k+1} = -p \cdot O_k + (1.1)R_k. \end{cases}$$

- (a) Use Maple to find the eigenvectors and eigenvalues of this system (which depend on p).
 - (b) Find values of p for which the origin is (a) an attractor; (b) a repellor; (c) a saddle point.
 - (c) For each case of stability (a),(b),(c) draw several trajectories for the dynamical system (plot the points of coordinates (R_k, O_k) when (R_1, O_1) given). Each case of stability should be illustrated on a different set of coordinates; plot the trajectories in different colors.
3. (15 points) The mean serum-creatinine level measured in 12 patients 24 hours after they received a newly proposed antibiotic was 1.2 mg/dL with a sample standard deviation (s) of 0.4mg/dL. If the mean of serum creatinine level in the general population is 1.0 mg/dL, using a significance level of 0.05, test if the mean serum-creatinine level in this group is higher from that of the general population. What is the p -value for the test?
4. (45 points) The data in the table below gives the infant-mortality rates per 1000 livebirths in the United States for the period 1960-1979 (x =year, y =infant mortality rate per 1,000 live births).

| | | | |
|------|------|------|------|
| x | y | x | y |
| 1960 | 26.0 | 1974 | 16.7 |
| 1965 | 24.7 | 1975 | 16.1 |
| 1970 | 20.0 | 1976 | 15.2 |
| 1971 | 19.1 | 1977 | 14.1 |
| 1972 | 18.5 | 1978 | 13.8 |
| 1973 | 17.7 | 1979 | 13.0 |

- (a) Fit a linear-regression line relating infant mortality rate to chronological year using these data.
- (b) Test for significance of the linear relationship developed in part (a).
- (c) If the trends continued for the next 10 years, then what would be the predicted infant-mortality rate in 1989?
- (d) Can the linear relationship developed in part (a) be expected to continue indefinitely? Why or why not?