398 Math in the City
Instructor: Petronela Radu
October 10, 2006

Homework 3

Due October 20

1. (25 points) Propose a nonlinear density-velocity relationship that satisfies the three assumptions given in class and for which you obtain the correct fundamental diagram of traffic (need to check that the function $q(\rho)$ is concave in ρ). What is the maximum traffic flow in this case? Is it better or worse than the value obtained in the linear case?

2. (35 points) Solve the following initial boundary value problem:

$$\begin{cases} 2\rho_t + 3\rho_x = 2e^{-t}, \quad x > 0, t > 0 \\ \rho(x,0) = 4x, \quad x > 0 \\ \rho(0,t) = 5 - t, \quad t > 0. \end{cases}$$

3. (35 points) Solve the pure initial value problem:

$$\begin{cases} u_t + 3xu_x - u = x, \quad x \in \mathbb{R}, t > 0 \\ u(x,0) = x^2, \quad x \in \mathbb{R}. \end{cases}$$

4. (30 points) Suppose initially ($t = 0$) the traffic density is $\rho = \rho_0 + \varepsilon \sin x$, where $|\varepsilon| << \rho_0$. Determine $\rho(x,t)$.

5. (25 points) Explain why a density wave moves forward for light traffic. Consider both cases in which the traffic ahead is getting heavier down the road and lighter.