398 Math in the City
Instructor: Petronela Radu
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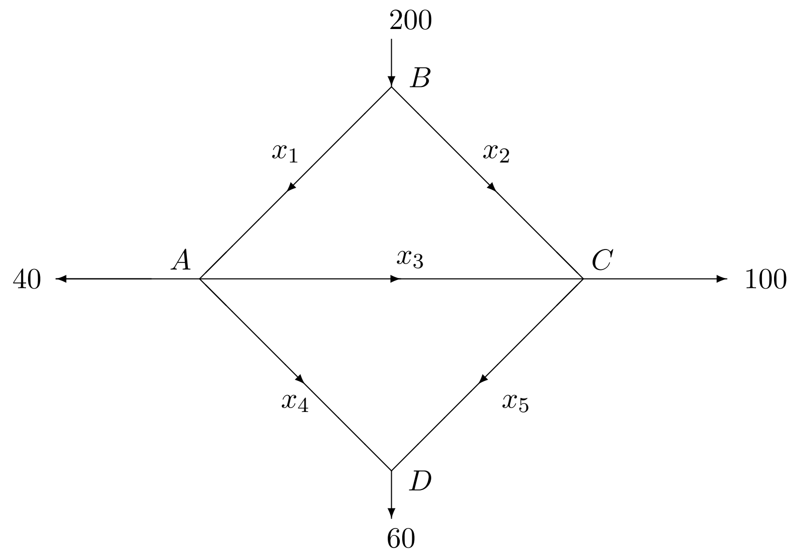
Homework 1

Due September 18

1. (30 points) What condition must be placed on a, b, and c so that the following system in unknowns x, y, and z has a solution?

$$\begin{cases} x + 2y - 3z = a \\ 2x + 6y - 11z = b \\ x - 2y + 7z = c \end{cases}$$

2. (a) (30 points) Find the general traffic pattern in the freeway network shown in the figure. (Flow rates are in cars/minute.)
   (b) Describe the general traffic pattern when the road whose flow is x_4 is closed.
   (c) When $x_4 = 0$, what is the minimum value of x_1?



3. (10 points) Explain why the matrix associated with a discrete dynamical system is different from the matrix of the system of differential equations which describes the *same* process. (Read the handout given in class about discrete dynamical systems). Can you tell how the two matrices are related?

4. (10 points) Show that if λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2.

5. (40 points) Consider the matrix

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

   (a) Find the eigenvalues of the matrix A.

(b) Find a basis for each of the eigenspaces of A.

(c) Write the characteristic equation for A and explain why A is diagonalizable.

(d) Diagonalize A.

6. (15 points) Find the matrix of the linear transformation that deforms the square $[0,4] \times [0,4]$ into the parallelogram with vertices at the points (0,0), (4,4), (2010,4), (2006,0). Find the area of the parallelogram by using the theorem about the determinant of the matrix of the linear transformation.

7. (15 points) Show that $t + 5$, $6 - t$, and $t^2 + 1$ form a basis for $\mathbb{P}_2$ (the set of polynomials of degree at most 2).