324 Introduction to PDEs
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Homework 7

Due November 3

Before writing your solutions commit yourself to being correct and clear throughout your arguments. In order to receive any credit the grader must be able to follow your arguments.

You may work on this assignment in groups of two students. Each student will receive the same grade, so make sure that the amount of work is equally shared by the team mates.

1. (20 points) Assume that at time $t = 0$ the temperature in a rod whose ends are in zero-degree temperature baths, is given by

$$T(x) = x(L - x), \quad 0 \leq x \leq L.$$

To find the temperature distribution in the rod for $t > 0$ by separation of variables, $T(x)$ must be expanded in a Fourier series. Find this series and discuss its convergence properties. Solve the IVP:

$$\begin{cases} u_t = u_{xx}, \quad 0 \leq x \leq L, t > 0 \\ u(0,t) = u(L,t) = 0, \quad t > 0 \\ u(x,0) = x(L-x). \end{cases}$$

2. (10 points) Use a CAS (Maple or Mathematica) to compute $s_1, s_2, ..., s_5$ (these are the partial Fourier sums) for the functions

   (a) $f(x) = e^x, \quad -2 \leq x < 2$

   (b) $f(x) = 4, -2 \leq x < 0$ and $f(x) = x^2, 0 \leq x < 2$.

   For each function sketch on the same plot (different colors) the function and its first five approximations.

3. (15 points) Problem 5 page 112.

4. (15 points) Problem 4 page 119.

5. (15 points) Problem 6 page 119.

6. (15 points) Problem 9 page 119.

7. (10 points) Write an essay about three different methods that you can use to solve the heat equation. What is your favorite method? Why? (The last two questions need to be answered by each person in the group.)