324 Introduction to PDEs
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Note on the convolution product

In class we defined the convolution product for functions $f, g : [0, \infty) \to \mathbb{R}$ by the formula:

$$(f * g)(t) = \int_0^t f(\tau) g(t - \tau) d\tau.$$

This is the formula that applies for the Laplace transform, since we usually apply this for the time variable which goes over the interval $[0, \infty)$.

In general, however, for functions $f, g : \mathbb{R} \to \mathbb{R}$ the convolution is given by

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau.$$

Please rectify your notes given in class, since I wrote incorrect bounds for the integral!

Note that if the functions f and g are defined only for positive numbers, then the only interval for τ where both quantities $f(\tau)$ and $g(t - \tau)$ are well defined is for $\tau \in (0, t)$.

Whenever you have to compute the convolution product for two functions defined on the entire axis, use the second formula.