Formula Sheet

The method of undetermined coefficients for $L[u] = g$

Case 1. If the characteristic value of g is not the characteristic value of L, take as a trial solution the most general family of functions having the same degree and the same characteristic value as g.

Case 2. Let $\gamma \pm i\omega$ be the characteristic value and k be the degree of the generalized exponential g. If $\gamma \pm i\omega$ is also a characteristic value of L with multiplicity m, take the trial solution of the form:

$$Y = t^m e^{\gamma t}[A(t)\cos\omega t + B(t)\sin\omega t],$$

where A, B are arbitrary polynomials of degree k.

The method of variation of parameters

For a first order linear equation $y' + py = g$ a **particular** solution can be taken of the form $y_1 u$, where y_1 is a solution of $y' + py = 0$ and u satisfies $y_1 u' = g$.

For a second order linear equation $y'' + py' + qy = g$, let $y_c = c_1 y_1 + c_2 y_2$ be the general solution of the homogeneous equation $y'' + py' + qy = 0$ and $W[y_1, y_2] = y_1 y_2' - y_1' y_2$ be its Wronskian. Then, a **particular** solution for $y'' + py' + qy = g$ is of the form $u_1 y_1 + u_2 y_2$, where

$$u_1' = \frac{-y_2 g}{W[y_1, y_2]}, \quad u_2' = \frac{y_1 g}{W[y_1, y_2]}.$$