

Practice Problems for Test 2

1. Verify that the given functions are solutions of the DE.

$$y''' + 2y'' - y' - 2y = 0, \quad e^t, e^{-t}, e^{-2t}.$$

Determine the Wronskian of each pair of two functions.

2. Find the general solution of the given DE

$$2y''' - 4y'' - 2y' + 4y = 0.$$

3. Let the linear differential operator L be defined by

$$L[y] = Ay^{(4)} - 5y^{(3)} + By'' + y,$$

where A, B are real constants.

- (a) Find $L[t^4]$.
(b) Find $L[e^{rt}]$.
(c) Write the characteristic equation for $L[y] = 0$.
4. Use the method of reduction of order to solve the DE:

$$(t-1)y'' - ty' + y = 0, \quad t > 1,$$

knowing that a particular solution is $y_1(t) = e^t$. (*Hint:* Use the substitution $y = y_1(t)v(t)$ and derive a DE for v .)

5. Find the solution of the initial value problem

$$u'' + u = F(t), \quad u(0) = 0, \quad u'(0) = 0,$$

where

$$F(t) = \begin{cases} F_0 t, & 0 \leq t \leq \pi, \\ F_0(2\pi - t), & \pi < t \leq 2\pi, \\ 0, & 2\pi < t. \end{cases}$$

Hint: Treat each time interval separately, and match the solutions in the different intervals by requiring that u and u' be continuous functions of t .

6. Use the method of variation of parameters to determine the solution of the given IVP:

$$y'' + y = \sec t; \quad y(0) = 2, \quad y'(0) = 1.$$

7. Use the method of undetermined coefficients to solve the following DEs:

(a) $2y'' + 3y' + y = t^2 + 3 \sin t$

(b) $y'' + 2y' + 5y = 4e^{-t} \cos 2t$

8. If an undamped spring-mass system with a mass that weighs 6lb and a spring constant 1lb/in is suddenly set in motion at $t = 0$ by an external force of $4 \cos t$ lb, determine the position of the mass at any time.

9. In the absence of damping the motion of a spring-mass system satisfies the initial value problem

$$mu'' + ku = 0, \quad u(0) = a, \quad u'(0) = b.$$

- (a) Show that the kinetic energy initially imparted to the mass is $mb^2/2$ and that the potential energy initially stored in the spring is $ka^2/2$, so that initially the total energy in the system is $(ka^2 + mb^2)/2$.
- (b) Solve the given initial value problem.
- (c) Using the solution in part b), determine the total energy in the system at any time. Your result should confirm the principle of conservation of energy for this system.