

Please Show Your Work To Ensure Full Credit.

1. (42 pts) Evaluate each of the following:

a. (8 pts) $\mathcal{L} [e^{-6t} \cos(2t)] (s)$

b. (10 pts) $\mathcal{L} [t \sin(3t)] (s)$

c. (12 pts) $\mathcal{L}^{-1} \left[\frac{s-1}{s^2+6s+45} \right]$

d. (12 pts) $\mathcal{L}^{-1} \left[e^{-\pi s} \frac{1}{(s-3)s} \right]$

2. (12 pts) Use the Laplace transform to solve the integral equation:

$$y(t) + 6 \int_0^t y(\tau) \sinh 3(t - \tau) d\tau = \cosh 3t.$$

3. (14 pts) Consider the system of ODE's:

$$\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y \end{cases} \quad (0.1)$$

Find **only one** solution to the system that corresponds to the positive eigenvalue of the matrix A associated with the system above.

4. (12 pts) Find $Y(s) = \mathcal{L}[y(t)](s)$ where $y(t)$ is the solution of the IVP:

$$y'' + 2y' + y = e^t U_3(t); \quad y(0) = -2, \quad y'(0) = 3.$$

You Need Not Find $y(t)$.

5. (20 pts) Consider the system

$$\begin{cases} \frac{dx}{dt} = -y(y-2) \\ \frac{dy}{dt} = (x-2)(y-2) \end{cases} \quad (0.2)$$

(a) (6 pts) Find all equilibrium solutions of the system (0.2) above.

(b) (2 pts) What is the solution $(x(t), y(t))$ that starts at $(x(0), y(0)) = (4, 2)$?

(b) (12 pts) Determine and describe all the trajectories of the system (0.2). Sketch the graph of only TWO trajectories (with orientation): one satisfies $(x(0), y(0)) = (2, 3)$, the second satisfies $(x(0), y(0)) = (2, -3)$; and briefly describe the behavior of each corresponding solution $(x(t), y(t))$ as t gets large.