MATH 221 **EXAM 2** **NAME:**______________________________

Fall 2006–Section 003 Instructor: Mohammad Rammaha

Please Show Your Work To Ensure Full Credit.

1. (12 points) Find the solution of the IVP:

$$2y'' + 5y' - 3y = 0, \quad y(0) = 7,\ y'(0) = 0.$$

2. (18 points) Use the method of undetermined coefficients (using annihilators) to find the general solution of the equation (Any other method you use will receive no credit):

$$y'' - 2y' + y = 7 + e^x$$

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3. (14 points) Write down the **Exact** form of the particular solution $y_p(x)$ of the equation

$$D^2(D+1)^2(D^2+4)y = 7x + \sin 2x$$

if the undetermined coefficient method is used. **You need not find the coefficients!!**

4. (12 points) Given that $y_1(x) = e^x$ is a solution to the equation $xy'' - (x+1)y' + y = 0$, for $x > 0$. Use the reduction of order formula to find a fundamental set for the equation, and write down its general solution. (Don't forget to rewrite the equation in normal form).

5. (18 points) Consider the ODE:

$$x^2y'' + 3xy' + y = \frac{2}{x}, \ x > 0. \tag{1}$$

Given that $\{y_1(x) = x^{-1}, y_2(x) = x^{-1}\ln x\}$ is a fundamental set for the corresponding homogeneous equation on $(0, \infty)$. Use the method of variation of parameters to find the general solution of the non-homogeneous equation (1).

6. (8 points) Find the annihilator of the function $3x^4 - x^7 - x^3 e^{-x} \sin 4x$.

7. (10 points) Find the general solution of the equation $y^{(4)} + 16y^{(2)} = 0$.

8. (8 points) Determine whether the following functions are linearly dependent or independent on $(-\infty, \infty)$. **Show a proof.**

$$f_1(x) = x, \quad f_2(x) = 4 - x, \quad f_3(x) = 4 + 2x.$$