MATH 221 **EXAM 1** **NAME:**____________________________

Section 003-Fall 2006 Instructor: Mohammad Rammaha

Please Show Your Work Neatly And Clearly To Ensure Full Credit.

1. (16 points) Solve the IVP: $y' - \frac{1}{x}y = 2\ln x, \quad x > 0, \ y(1) = 7.$

2. (14 points) Find an EXPLICIT FORMULA for the solution of the IVP: $(1+x^2)\frac{dy}{dx} = y^2, \ y(0) = 1$; and from the formula determine the domain of the solution.

3. (12 points) Consider the IVP:

$$(\cos x)\ y' + x^5 e^{x^2}\ y = 0, \quad y(0) = 17. \tag{1}$$

(a) (10 points) Find the largest open interval on which the following IVP (1) has a unique solution.

(b) (2 points) If the initial condition in (1) is changed to the condition $y(0) = 0$, can you guess (with justification) the solution to the new IVP?

4. (10 points) Describe and carefully sketch the region in the $xy-$plane where the hypotheses of Picard's existence and uniqueness theorem are satisfied (so that there is a unique solution through each given initial point (x_0, y_0) in this region).
$y' = x^5 + \sqrt{y - x^2}, \quad y(x_0) = y_0.$

5. (12 points) Use the Euler method with step size $h = \frac{1}{2}$ to approximate the value of $y(\frac{3}{2})$; where $y(x)$ is the solution of the IVP:

$$\frac{dy}{dx} = 1 + x^2 + y^2, \quad y(0) = 0. \tag{2}$$

Show the details of your work by using a proper table.

6. (8 points) The following ODE is not linear nor separable:

$$\frac{dy}{dx} = (x + y + 9)^4.$$

Use an appropriate change of variables to reduce it into a separable equation, BUT DON'T SOLVE the resulting equation.

7. (28 points) Let $y(t)$ be the population of rabbits in a small area of a forrest and time t is measured in months. Assume the $y(t)$ obeys the logistic model: $\frac{dy}{dt} = \frac{1}{10}y(50 - y)$. However, the foxes in the same area of the forrest feed on the rabbits at the rate of 60 rabbits per month. Assume $y(0) = y_0$ is the initial population.
(a) (8 points) Set up (BUT DON'T SOLVE) the initial value problem whose solution gives $y(t)$ the population of the rabbits.

(b) (8 points) Find all equilibrium solutions to the ODE you found in part (a) and sketch the phase line diagram. Classify the equilibrium solutions as sink (stable), source (unstable), or node (semi-stable).

(c) (6 points) On the same set of axis, sketch the solutions $y_1(t)$, $y_2(t)$ of the IVP in part (a) with the initial conditions $y_1(0) = 10$, $y_2(0) = 25$.

(d) (6 points) For what values of y_0 can you be certain that the population of rabbits won't go to zero as $t \to \infty$? That is, the foxes will never go hungry.