**MATH 221H- PROJECT #1**
**BUGS IN MUTUAL PURSUIT**

Four identical bugs start moving at the same time on a flat table, each at the same constant speed of 16 cm/min. Assume that initially bugs 1 through 4 are located at the points $(1,1),(-1,1),(-1,-1),(1,-1)$, respectively. Assume that the units in the $xy$-plane are measured in meters. Also assume that bug 1 always heads directly towards bug 2, bug 2 towards bug 3, bug 3 towards bug 4, and bug 4 towards bug 1, so their paths are mutually congruent. Let $(x,y)$ denote the location of each bug at time $t$.

1. Use the symmetry of the paths to show that at any moment the bugs are the corners of a square, and in particular, if bug 1 is at $(x,y)$, then bugs 2, 3,4 are respectively at $(-y,x),(-x,-y),(y,-x)$.
2. Using the notation of part 1, show that the paths of bugs 1 and 3 satisfy the differential equation $\frac{dy}{dx}=\frac{y-x}{x+y}$, whenever $x+y\neq 0$. What is the analogous differential equation for bugs 2 and 4?
3. Use Maple to sketch the slope field diagram of the ODE in part 2. On this diagram, sketch a reasonable curve for the path of bug 1.
4. Show that the path of bug 1 is the curve given implicitly by
$$ln\sqrt{x^2+y^2}+arctan(y/x)=C_1,$$
where $C_1$ is a constant that you should find. *Hint: here you have to solve the ODE in part 2. But this ODE is not linear nor separable. To find its general solution, use the change of variables $y=xu$ to reduce the ODE into a separable equation for $u$.*
5. By passing to polar coordinates $x=rcos\theta,\ y=rsin\theta$, show that the path of bug 1 is given by the polar equation $r=Ce^{-\theta}$, where $C$ is a constant that you should find. This polar curve is called a **logarithmic spiral**. Sketch it by hand, by using your calculator, or by using Maple.
6. Recall that, if $\mathcal{C}$ is a smooth polar curve given by
$\mathcal{C}: r=f(\theta),\ \theta_0\leq\theta\leq\theta_1$, then the arc-length of $\mathcal{C}$ is given by:
$\int_{\theta_0}^{\theta_1}\sqrt{r^2+(\frac{dr}{d\theta})^2}d\theta$. Use this fact to show that all four bugs reach the origin at the same time $T_0$ and find the exact value of $T_0$.
7. Find the time $T$ at which bug 1 is 2 cm from bug 2.
8. How many times does bug 1 wind around the origin during the first 12.499 minutes? Discuss the motion of bug 1 on the time interval $12.499<t\leq T_0$.