Math 107-Sec 250 **Exam 3** **Name:** _____

Spring 2004 **Recitation Instructor:** _____

No.	1	2	3	4	5	6	7	Total
score								

1. (20 points, 10 points each) Determine whether the following series converge absolutely, converge conditionally or diverge. You must show all details to receive credit.

   a. $\sum_{k=1}^{\infty}(-1)^k \frac{k}{2k^3+1}$

   b. $\sum_{k=1}^{\infty}(-1)^k \frac{2^k}{(3k)!}$

2. (16 points, 8 points each) Determine whether the following series are convergent or divergent. If the series is convergent, **find its sum.**

   a. $-\frac{2}{3}+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^3+\left(\frac{2}{3}\right)^4-\ldots$

   b. $\sum_{k=1}^{\infty}(-1)^k\frac{k}{2k+1}$

3. (9 points) Find the **first three non-zero terms** of the Taylor series of $f(x)=(1-x)^{1/2}$ about $x=0$. You need not find every term in the series.

4. (10 points) By using the Taylor series: $\frac{1}{1-x}=\sum_{k=0}^{\infty}x^k$, $|x|<1$; find the Taylor series of $f(x)=\frac{x}{3+2x}$ about $x=0$. Make sure to include the interval of convergence.

5. (18 points) Find the interval on which the following power series converges absolutely. Also, find the radius of convergence and make sure to **discuss in details** the convergence/divergence of the series at the end points of the interval you have found.

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} (x-1)^{k+2} \tag{1}$$

6. (12 points) Suppose that a function $f(x)$ has the following Taylor series about $x = 0$:

$$f(x) = \sum_{k=17}^{\infty} \frac{(-1)^k}{8^k} x^{3k}, \quad -2 < x < 2. \tag{2}$$

Find the exact values of $f^{(9)}(0)$ and $f^{(99)}(0)$.

7. (15 points) Let $\mathcal{C}$ be a plane curve given by $\mathcal{C} : x = t^5 + t, \ y = 1 + t\sin(t-2), \ 0 \leq t \leq 7$.

a. (10 pts.) Find the equation of the tangent line to $\mathcal{C}$ at the point that corresponds to $t = 2$.

b. (5 pts.) Find **but don't evaluate** a definite integral whose value gives the arc-length of $\mathcal{C}$.