

No.	1	2	3	4	5	6	Total
score							

Important Note: On this exam and future exams as well, your work must be shown. Straight answers without work that support them will receive **zero points**; even if they were correct.

1. (27 points, 9 points each) Evaluate each of the following integrals (**Please read the Important Note above. Here, no calculators allowed**).

a. $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

b. $\int \frac{x}{\sqrt{x-1}} dx$

c. $\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$

2. (14 points, 7 points each) Let R be the **bounded** region enclosed by $y = f(x) = 2x + 1$, $y = g(x) = x^2 - 2$.
- a. Find all points of intersection between f and g and carefully sketch the region R .

b. Find (**but don't evaluate**) an integral whose value gives the exact area of the region R .

3. (11 points) A cold drink is poured out at $50^\circ F$. After 3 minutes of sitting in a $70^\circ F$ room, the temperature of the drink has risen to $57^\circ F$. Find the drink's temperature $T(t)$ at any time t , assuming that $T(t)$ obeys Newton's law of heating and cooling: $\frac{dT}{dt} = k(T - a)$.

4. (16 points, 8 points each) Let Ω be the region enclosed by $y = x^3$, $y = x$ from $x = 0$ to $x = 1$.
- a. Find (**but don't evaluate**) an integral whose value gives the volume of the solid obtained by revolving the region Ω about the **x-axis**, **By using the method of slicing**.

- b. Find (**but don't evaluate**) an integral whose value gives the volume of the solid obtained by revolving the region Ω about the **y-axis**, **By using the method of cylindrical shells**.

5. (16 points, 8 points each) A water tank is in the form of a right circular cylinder with height 10 ft and radius 6 ft. Assume that the tank is filled with water weighing $\rho = 62.4 \text{ lb}/\text{ft}^3$.
- Find a Riemann sum whose value approximates the work required to pump all of the water over the top of the tank.

b. Write down **but do not evaluate** an integral whose value is exactly the work required to pump all of the water over the top of the tank.

6. (16 points, 8 points each) A rod of length 2 meters is placed on the x -axis from $x = 0$ to $x = 2$. Assume its density $\rho(x) = 3 - e^{-4x}$ kilograms per meter; $x \in [0, 2]$.
- Find a Riemann sum whose value approximates the mass of the rod.

b. Write down **but do not evaluate** an integral whose value is exactly the mass of the rod.