

# FINAL EXAM

Math 106, Fall Semester 2008

Name (Print): \_\_\_\_\_

Student ID Numer: \_\_\_\_\_

TA Name: \_\_\_\_\_

Please Circle your professor's name:

B. Deng

K. Fey

S. Iyengar

M. Rammaha

B. Rogge

Please Circle your class time:

7:30 a.m.

8:30 a.m.

9:30 a.m.

10:30 a.m.

11:30 a.m.

12:30 p.m

6:30 p.m.

## INSTRUCTIONS:

- *There are 7 pages of questions and this cover sheet.*
- *SHOW ALL YOUR WORK. Partial credit will be given only if your work is relevant and correct.*
- *This examination is closed book. Calculators that perform symbolic manipulations such as the TI-89, TI-92 or their equivalence, are **not permitted**. Other calculators may be used. Turn off and put away all cell phones.*

---

Question	Points	Score
1	12	
2	24	
3	14	
4	16	
5	12	
6	16	
7	10	
8	18	
9	8	
10	12	
11	10	
12	18	
13	30	
Total	200	

1. [12 Points] By using the limit definition of the derivative, find  $f'(1)$  if  $f(x) = \sqrt{x+3}$ . Other methods for finding the derivative will not receive credit. Show all your work.

2. [24 Points] Evaluate each of the following: (Credit will be given only if you show work that justifies your answer.)

a) [8 Points]  $\int_0^1 (4x^3 + e^{2x} - 1) dx$ . (Decimal approximations such as 2.1234 will not get credit.)

b) [8 Points]  $\int \left( \frac{1}{4x+1} + \sin(4x+1) \right) dx$ .

c) [8 Points]  $\int \frac{1}{\sqrt{x}} \sec^2(\sqrt{x}) dx$ .

3. [14 Points] Find the equation of the tangent line to the curve  $xe^{y-1} + x^3 = y^3 + 9$  at the point  $(2, 1)$ .

4. [16 Points] Find:

a) [8 Points]  $\frac{d}{dx}F(x)$ , where  $F(x) = \int_1^{x^2} (1+t)^t dt$ .

b) [8 Points]  $f(x)$ , if  $f'(x) = \frac{1}{\sqrt{x}} + \frac{1}{1+x^2}$  and  $f(1) = 2$ .

5. [12 Points] Find the **exact** value of the following limit:  $\lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{x^2}$ . (Show work that justifies your answer. Numerical and/or graphical reasoning is not sufficient and will receive no credit.)

6. [16 Points] Find  $\frac{dy}{dx}$  for each of the following (Show work that justifies your answer. DO NOT SIMPLIFY):

a) [8 Points]  $y = \frac{\ln|x^3 + 1|}{4^x + \cos x}$ .

b) [8 Points]  $y = (1 + x \sin^{-1}(3x))^5$ .

7. [10 Points] A spherical balloon is being inflated with air at the rate of 2 cubic feet per minute. At what rate is the radius of the balloon increasing when the radius is 5 feet? Hint: the volume enclosed by a spherical balloon of radius  $r$  is  $\frac{4\pi}{3}r^3$ .

8. [18 Points] Let  $\mathbf{R}$  be the region in the first quadrant bounded by the graphs of  $y = x^3$ ,  $x = 1$  and  $y = 0$ . Let  $\mathbf{S}$  be the solid obtained by revolving  $\mathbf{R}$  about the **y-axis**.

a) [9 Points] By using the method of slicing (taking cross sections perpendicular to the y-axis), find, **but don't evaluate**, a definite integral whose value gives the volume of  $\mathbf{S}$ .

b) [9 Points] By using the method of cylindrical shells, find, **but don't evaluate**, a definite integral whose value gives the volume of  $\mathbf{S}$ .

9. [8 Points] The inverse function,  $f^{-1}$ , of the function  $f(x) = x^3 + 2x - 7$  exists (You need not show this, **and** don't find  $f^{-1}$ ). Find the derivative of  $f^{-1}(x)$  at the point  $(5,2)$ .

10. [12 Points] This problem has two independent parts:

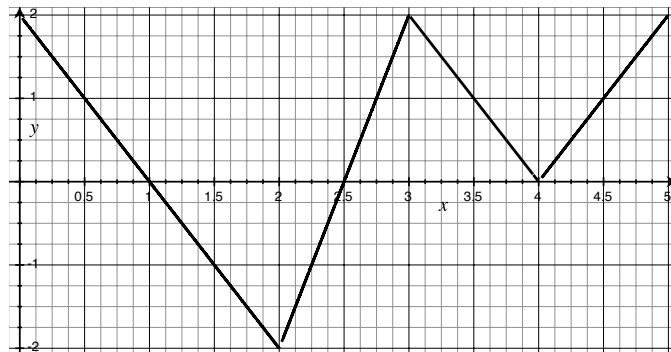
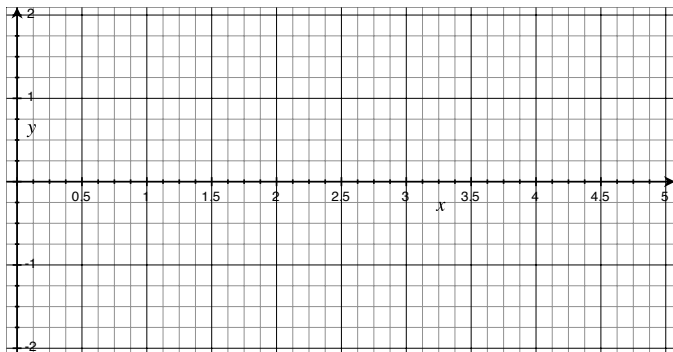
a) [6 Points] Let  $f(x)$  be a differentiable function whose tangent line at  $x = 1$  is given by  $5x + y = 8$ . What are  $f(1)$  and  $f'(1)$ ?

b) [6 Points] Let  $g(x)$  be a differentiable function with  $g(3) = -2$  and  $g'(3) = 4$ . If  $P(x) = g(\sqrt{x^2 + 8})$ , find  $P'(1)$ .

11. [10 Points] Find, **but don't evaluate**, a definite integral whose value gives the area of the bounded region enclosed by the graphs of  $y = -x$  and  $y = 2 - x^2$ . (make sure to sketch the region).

12. [18 Points] A farmer wants to fence a rectangular region adjacent to a straight river. The area of the region is  $125 m^2$ . The cost of fencing is \$5 per meter for the side parallel to the river, \$4 per meter for the sides perpendicular to the river, and no fencing is needed along the river. Find the dimensions of the rectangular region that minimizes the cost of fencing.

13. [30 Points] Let  $f(x)$  be a continuous function on  $[0, 5]$  with  $f(0) = 0$ , and whose **derivative**  $f'(x)$  is as shown below.



a) [9 Points] Find the  $x$ -coordinates of all critical points of  $f$  in the interval  $[0, 5]$  and classify them as local maximum, local minimum, or neither.

b) [9 Points] List all inflection points of  $f$  and all intervals on which  $f$  is concave up and concave down.

c) [6 Points] Find  $f(5)$ .

d) [6 Points] Sketch a reasonable, **but correct** graph of  $y = f(x)$  in the empty plot next to the graph of  $f'(x)$  on  $[0, 5]$ . Make sure to highlight all important features of the graph of  $y = f(x)$ .