

**You must show all of your work to receive full credit!**

No.	1	2	3	4	5	6	Total
score							

- (1) (8 points) Suppose that  $f$  is a continuous function on  $\mathbb{R}$  with  $f(7) = 1$  and  $f'(7) = \frac{3}{2}$ . Use linear approximation to estimate  $f(7.04)$ .

- (2) (12 points) Let  $f(x) = x^3 + 4x - 7$ ,  $x \in \mathbb{R}$ .

(a) (4 points) Show that its inverse function  $f^{-1}$  exists.

(b) (8 points) Find the equation of the tangent line to  $y = f^{-1}(x)$  at the point  $(9, 2)$ .

(3) (24 points, 8 points each) Find  $\frac{dy}{dx}$  if (You need not simplify):

(a)  $y = x^3 \arctan(\sqrt{x})$

(b)  $y = (\sin^{-1}(4x) + \ln |1 - \cos x|)^9$

(c)  $x^3 e^y + \sec^{-1} y = 9x$

(4) (8 points) Assume that an infected area of injury is a square. Without medical treatment, the infected area started to increase at the rate of  $0.8 \text{ cm}^2/\text{day}$ . At what rate is the length of the infected area increasing when the length is  $4 \text{ cm}$ ?

(5) (24 points) This question deals with the function  $f(x) = 3x^5 - 5x^3 + 1$ .

(a) (6 points) Find all critical points of  $f$ .

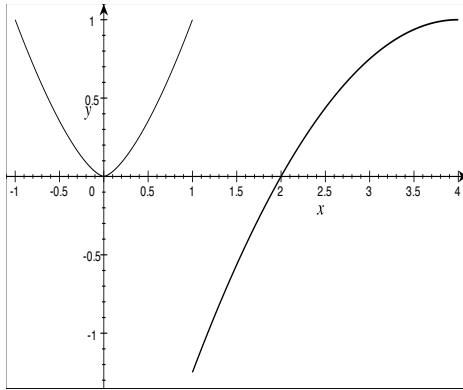
(b) (6 points) Find the global (absolute) maximum and minimum values of  $f$  on the interval  $[-\frac{1}{2}, 2]$ .

(c) (8 points) Find all inflection points of  $f$  and determine the open intervals on which  $f$  is concave up, concave down.

(d) (4 points) Find the linearization of  $f$  at  $x = 2$ .

(6) (24 points)

Given that  $y = f(x)$  is a continuous function on the interval  $[-1, 4]$  with  $f(0) = 0$  and whose **derivative function**  $y = f'(x)$  is as shown below



(a) (9 points) **Find** and **classify** all of the critical points for the function  $f(x)$  in the interval  $[-1, 4]$ .

(b) (8 points) Determine the intervals on which  $f$  is concave up and down, and list all inflection points for the function  $f(x)$ .

(c) (7 points) In the space next to the graph of  $y = f'(x)$ , sketch a reasonable but **correct** graph of  $y = f(x)$ . Make sure to highlight all important features of the graph.