

Some “Tricks” for Integration

Trick	Examples
Expand	$\int (1 + e^x)^2 dx = \int (1 + 2e^x + e^{2x}) dx = x + 2e^x + \frac{1}{2}e^{2x} + C$
Split Fractions	$\int \frac{1+x}{x^2+1} dx = \int \left(\frac{1}{x^2+1} + \frac{x}{x^2+1} \right) dx$ $= \int \frac{1}{x^2+1} dx + \frac{1}{2} \int \frac{1}{u} du \quad \boxed{\text{substitute: } \begin{matrix} u = x^2 + 1 \\ du = 2x dx \end{matrix}}$ $= \tan^{-1} x + \frac{1}{2} \ln u + C = \tan^{-1} x + \frac{1}{2} \ln (x^2 + 1) + C$
Add and Subtract an Expression	$\int \frac{2x}{x^2 + 2x + 1} dx = \int \frac{2x + 2 - 2}{x^2 + 2x + 1} dx \quad \boxed{\text{add and subtract 2}}$ $= \int \frac{2x + 2}{x^2 + 2x + 1} dx - 2 \int \frac{1}{(x + 1)^2} dx \quad \boxed{\text{split the fraction}}$ $= \int \frac{1}{u} du - 2 \int \frac{1}{v^2} dv \quad \boxed{\begin{matrix} u = x^2 + 2x + 1 \\ du = (2x + 2)dx \\ \text{substitute: } \quad \text{and} \\ v = x + 1 \\ dv = dx \end{matrix}}$ $= \ln u + \frac{2}{v} + C = \ln (x^2 + 2x + 1) + \frac{2}{x + 1} + C$
Example 1:	$\int \frac{1}{\sqrt{2x - x^2}} dx = \int \frac{1}{\sqrt{1 - (x^2 - 2x + 1)}} dx \quad \boxed{\text{add and subtract 1 to complete the square}}$ $= \int \frac{1}{\sqrt{1 - (x - 1)^2}} dx$ $= \int \frac{1}{\sqrt{1 - u^2}} du \quad \boxed{\text{substitute: } \begin{matrix} u = x - 1 \\ du = dx \end{matrix}}$ $= \sin^{-1} u + C = \sin^{-1}(x - 1) + C$
Example 2:	$\int \frac{1}{4x^2 - 2x + 9} dx = \int \frac{1}{(4x^2 - 2x + \frac{1}{4}) - \frac{1}{4} + 9} dx \quad \boxed{\text{add and subtract } \frac{1}{4} \text{ to complete the square}}$ $= \int \frac{1}{(2x - \frac{1}{2})^2 + \frac{35}{4}} dx$ $= \frac{4}{35} \int \frac{1}{\left(\frac{4}{\sqrt{35}}x - \frac{1}{\sqrt{35}}\right)^2 + 1} dx \quad \boxed{\text{substitute: } \begin{matrix} u = \frac{4}{\sqrt{35}}x - \frac{1}{\sqrt{35}} \\ du = \frac{4}{\sqrt{35}} dx \end{matrix}}$ $= \frac{1}{\sqrt{35}} \int \frac{1}{u^2 + 1} du = \frac{1}{\sqrt{35}} \tan^{-1} u + C$ $= \frac{1}{\sqrt{35}} \tan^{-1} \left(\frac{4}{\sqrt{35}}x - \frac{1}{\sqrt{35}} \right) + C$
Complete the Square	

Trick	Examples
Divide Improper Rational Fractions	$\int \frac{x^2}{x^2+1} dx = \int \left(1 - \frac{1}{x^2+1}\right) dx$ $= x - \tan^{-1} x + C$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">long division or synthetic division</div>
Trig Identities	$\int \cos^2 x dx = \int \left(\frac{1}{2} + \frac{1}{2} \cos 2x\right) dx$ $= \frac{1}{2}x + \frac{1}{4} \sin 2x + C$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">half-angle formula: $\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$</div>
Multiply and Divide	$\int \frac{1}{1+\sin x} dx = \int \frac{1}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x} dx$ $= \int \frac{1-\sin x}{1-\sin^2 x} dx$ $= \int \frac{1-\sin x}{\cos^2 x} dx$ $= \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x}{\cos^2 x} dx$ $= \int \sec^2 x dx - \int \tan x \sec x dx$ $= \tan x - \sec x + C$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">multiply and divide by $1 - \sin x$</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">expand denominator</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Pythagorean's Theorem: $\cos^2 \theta + \sin^2 \theta = 1$</div> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">split the fraction</div>