(*) 33. Find examples of spaces and subspaces \( A_0 \subseteq X_0 \) and \( A_1 \subseteq X_1 \) so that \( H_*(X_0) \cong H_*(X_1) \) and \( H_*(A_0) \cong H_*(A_1) \), but \( H_*(X_0, A_0) \not\cong H_*(X_1, A_1) \). (If you want to make it more challenging, find examples with all of the spaces path-connected? Note that Problem #35 gives a hint on how not to solve this problem...)

34. Show that if \( A \subseteq X \) and the identity map \( I : X \to X \) is homotopic to a map \( f : X \to X \) with \( f(X) \subseteq A \), then for every \( n \), \( H_n(A) \cong H_n(X) \oplus H_{n+1}(X, A) \).

(\( \text{So } A \text{ has more "holes" than } X \text{ does...} \))

35. (a): Let \( f : (X, A) \to (Y, B) \) be a map of pairs such that both \( f : X \to Y \) and \( f : A \to B \) are homotopy equivalences. Show that the induced map \( f_* : H_n(X, A) \to H_n(Y, B) \) is an isomorphism for all \( n \).

(b): Show that the inclusion map \( \iota : (D^n, \partial D^n) \to (D^n, D^n \setminus \{0\}) \) satisfies the hypotheses of (a), but is \textit{not} a homotopy of pairs, that is, there is \textit{not} a map \( f : (D^n, D^n \setminus \{0\}) \to (D^n, \partial D^n) \) so that \( f \circ \iota \) and \( \iota \circ f \) are both homotopic, as maps of pairs, to the identity maps.

36. Compute the singular homology groups of the pseudo-projective planes \( P_n, n \geq 2 \), shown below, where the boundary has been subdivided into \( n \) equal arcs.

\[
\begin{array}{c}
\text{a} \\
\text{a}
\end{array}
\]

\( \text{a} \)

\( \text{a} \)

37. For a space \( X \) the \textit{cone} on \( X \) is the quotient space
\[ cX = X \times I / \{(x, 0) \sim (y, 0) : x, y \in X \} = X \times I / X \times \{0\}, \]
and the \textit{suspension} of \( X \) is the quotient space 
\[ SX = X \times I / \{(x, 0) \sim (y, 0), (x, 1) \sim (y, 1) : x, y \in X \}, \]
which can be thought of as two cones on \( X \) glued along their common copy of \( X \). Show that for any path connected space \( X \), \( \tilde{H}_i(cX) = 0 \) and \( \tilde{H}_i(SX) \cong \tilde{H}_{i-1}(X) \) for all \( i \).

38. Show that, for any collection of finitely generated abelian groups \( G_1, \ldots, G_n \), there is a path-connected space \( X \) with \( \tilde{H}_i(X) \cong G_i \) for all \( i = 1, \ldots, n \) and \( \tilde{H}_i(X) = 0 \) for all other \( i \).