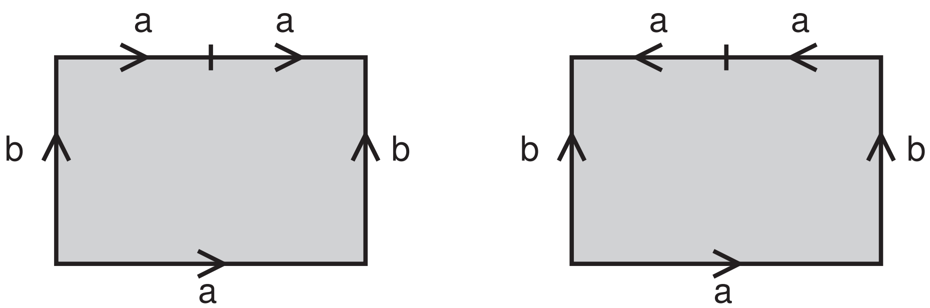
Math 872 Algebraic Topology

Problem Set # 5

Starred (*) problems due Thursday, March 8

22. Find a Δ-complex structure for, and compute the (simplicial) homology groups of, the space obtained from an annulus $A = S^1 \times I$ by gluing $S^1 \times \{1\}$ to $S^1 \times \{0\}$ by a map representing 2 times the generator of $\pi_1(S^1)$. (See figure below.)



(*) 23. Find a Δ-complex structure for, and compute the (simplicial) homology groups of, the space obtained from an annulus $A = S^1 \times I$ by gluing $s^1 \times \{1\}$ to $s^1 \times \{0\}$ by a map representing -2 times the generator of $\pi_1(S^1)$. (See figure above.)

(*) 24. Compute the simplicial homology groups of the 3-simplex Δ^3 (that is, the Δ-complex obtained from 4 vertices by gluing on 6 1-simplices, 4 2-simplices and a single 3-simplex in the "obvious" way).

25. Regarding the n-simplex $X = \Delta^n$ as a Δ-complex in the natural way, show that if $A \subseteq X$ is a subcomplex with $H_{n-1}(A) \neq 0$, then $A = \partial\Delta^n$. (Hint: show that an $(n-1)$-cycle for A (and hence for X) must either be 0 or have non-zero coefficient for <u>every</u> $(n-1)$-dimensional face of X.)

26. Show that if $A \subseteq X$ is a retract of X, then the inclusion map $\iota : A \to X$ induces an injection on all singular homology groups.