11. Compute the fundamental group of the complement $X$ of the coordinate axes in $\mathbb{R}^3$, i.e., $X = \{(x, y, z) \in \mathbb{R}^3 : |xy| + |xz| + |yz| \neq 0\}$. (Hint: Find a simpler space to compute the fundamental group of.)

12. Show that if $p_1 : \tilde{X}_1 \to X_1$ and $p_2 : \tilde{X}_2 \to X_2$ are covering maps, then the map $p = p_1 \times p_2 : \tilde{X}_1 \times \tilde{X}_2 \to X_1 \times X_2$, given by $p(x_1, x_2) = (p_1(x_1), p_2(x_2))$, is also a covering map.

(*) 13. Let $Y \subseteq \mathbb{R}^2$ denote the quasi-circle, given by $Y = \{(x, \sin(1/x)) : x \in (0, \frac{1}{\pi}] \cup \{0\} \times [-1, 1] \cup [-\frac{1}{\pi}, 0] \times \{0\} \cup \{\left(\frac{1}{\pi} \cos(t), -\frac{1}{\pi} \sin(t)\right) : 0 \leq t \leq \pi\}$ (See Hatcher, p.79, problem # 7 for an (approximate) picture.) The quotient map $q : Y \to Y/\sim$ that collapses the vertical subinterval $\{0\} \times [-1, 1]$ to a point gives a space homeomorphic to $S^1$. Show that $\pi_1(Y) = \{1\}$ (hint: show that any path in $Y$ is disjoint from $\{(x, y) \in Y : 0 < x < \epsilon\}$ for some $\epsilon > 0$), but the map $q$ does not lift to the universal covering $p : \mathbb{R} \to S^1$. [This demonstrates that we cannot in general eliminate the hypothesis that $Y$ be locally path-connected, in the lifting criterion.]

14. Let $p : \tilde{X} \to X$ be a finite-sheeted covering space, with $\tilde{X} \neq \emptyset$. Show that $\tilde{X}$ is compact and Hausdorff $\iff$ $X$ is compact and Hausdorff.

(*) 15. Let $p : \tilde{X} \to X$ and $q : \tilde{Y} \to Y$ be covering spaces of path-connected, locally path connected spaces $X$ and $Y$ with $\tilde{X}$ and $\tilde{Y}$ simply-connected. Show that if $X$ and $Y$ are homotopy equivalent, then $\tilde{X}$ and $\tilde{Y}$ are homotopy equivalent. [Problem 6 from Problem Set # 1 might help.]