Math 872 Algebraic Topology

Homework # 2

Starred (*) problems due Thursday, February 1

6. Show that if there are maps $f : X \to Y$ and $g, h : Y \to X$ such that $f \circ g : Y \to Y$ and $h \circ f : X \to X$ are homotopic to I_Y and I_X, respectively, then f is a homotopy equivalence. Show that the same is true if $f \circ g$ and $h \circ f$ are homotopy equivalences.

(*) 7. Show that if X is a CW complex, $A, B \subseteq X$ are subcomplexes, $A \cup B = X$, and A, B and $A \cap B$ are contractible, then X is contractible.

8. Show that if $f : \partial D^n \to X$ is the attaching map of an n-cell D^n, with $n \geq 3$, then the inclusion $X \hookrightarrow X \cup_f D^n$ induces an isomorphism on π_1. Show that the same is true if we attach any (finite or infinite) collection of $n \geq 3$ cells.

(*) 9. Let $X =$ the space obtained from a cube $J^3 = J \times J \times J$, $J = [-1, 1]$, by gluing opposite square faces to one another with a 90-degree righthand twist (e.g., glue $J \times J \times \{-1\}$ to $J \times J \times \{1\}$ by the map $(x, y, -1) \mapsto (y, -x, 1)$. Describe a CW structure for X and compute a presentation for $\pi_1(X)$.

10. Starting with a 2-disk, D^2, with two small sub-disks deleted, show that there are essentially only two spaces obtained by identifying each of the two interior boundary circles to ∂D^2 by homeomorphisms. Compute presentations for the fondamental groups of each, and (by abelianizing) show that the two groups are not isomorphic, so the two spaces are not homotopy equivalent.