6. Show that if there are maps $f : X \to Y$ and $g, h : Y \to X$ such that $f \circ g : Y \to Y$ and $h \circ f : X \to X$ are homotopic to $I_Y$ and $I_X$, respectively, then $f$ is a homotopy equivalence. Show that the same is true if $f \circ g$ and $h \circ f$ are homotopy equivalences.

(*) 7. Show that if $X$ is a CW complex, $A, B \subseteq X$ are subcomplexes, $A \cup B = X$, and $A, B$ and $A \cap B$ are contractible, then $X$ is contractible.

8. Show that if $f : \partial D^n \to X$ is the attaching map of an $n$-cell $D^n$, with $n \geq 3$, then the inclusion $X \hookrightarrow X \cup_f D^n$ induces an isomorphism on $\pi_1$. Show that the same is true if we attach any (finite or infinite) collection of $n \geq 3$ cells.

(*) 9. Let $X = \text{the space obtained from a cube } J^3 = J \times J \times J, J = [-1, 1], \text{ by gluing opposite square faces to one another with a 90-degree righthand twist (e.g., glue } J \times J \times \{-1\} \text{ to } J \times J \times \{1\} \text{ by the map } (x, y, -1) \mapsto (y, -x, 1). \text{ Describe a CW structure for } X \text{ and compute a presentation for } \pi_1(X).

10. Starting with a 2-disk, $D^2$, with two small sub-disks deleted, show that there are essentially only two spaces obtained by identifying each of the two interior boundary circles to $\partial D^2$ by homeomorphisms. Compute presentations for the fundamental groups of each, and (by abelianizing) show that the two groups are not isomorphic, so the two spaces are not homotopy equivalent.