Math 856 Homework 6

Starred (*) problems to be handed in Tuesday, November 28

(*) 30: (Lee, p.172, Problem 7-8) If $F : M \to N$ is a submersion and X is a smooth vector field on N, show that there is a smooth vector field Y on M that is F-related to X.

31: Show that the set $\{(x, |x|) : x \in \mathbb{R}\}$ is not the image of any immersion of $\mathbb{R}$ into $\mathbb{R}^2$. (Hint: nothing fancy, just beat it over the head with calculus?)

(*) 32: (a) If $U \subseteq \mathbb{R}^n$ is open and $F : U \to \mathbb{R}^m$ is smooth, show that the graph of F, $\Gamma(F) = \{(x, F(x)) \in \mathbb{R}^{n+m} : x \in U\}$ is an embedded submanifold of $\mathbb{R}^{n+m}$.

(b) Show, conversely, that every embedded submanifold of $\mathbb{R}^N$ is locally of this form. (You will need to use the implicit function theorem?)

33: (a) Show that an immersion from one n-manifold to another is an open map.

(b) Show that if M and N are n-manifolds, M is compact, N is connected, and $F : M \to N$ is an immersion, then F is onto.

34: If $S \subseteq M$ is a closed, embedded submanifold, $U \supseteq S$ is an open neighborhood of S, and $f : S \to \mathbb{R}$ is a smooth function, show that there is a smooth function $F : M \to \mathbb{R}$ with $F|_S = f$ and $\text{supp}(F) \subseteq U$.