Math 445 Homework 5
Due Friday, October 10

17. Show that if \(a^2 + 2b^2 = c^2\) and \(p|(a, c)\) for some prime \(p\), then \(p|b\).
   (Note: \(p = 2\) is treated differently...)

18. Show that if \(a^2 + 2b^2 = c^2\) and \((a, c) = 1\), then \(a\) and \(c\) are odd, and \(b\) is even.

19. Continuing the analysis: with \((a, c) = 1\), writing
   \[2b^2 = c^2 - a^2 = (c - a)(c + a) = (2u)(2v),\]
   show that \((u, v) = 1\), one of \(u, v\) is odd and the other even, and \(u = 2x^2, v = y^2\) or \(u = y^2, v = 2x^2\) for some integers \(x, y\).

20. Finishing the analysis: show that \((2x^2 - y^2)^2 + 2(2xy)^2 = (2x^2 + y^2)^2\), and every solution to \(a^2 + 2b^2 = c^2\) with \(a, b, c \in \mathbb{N}\) has this form:
   \[a = |2x^2 - y^2|, b = 2xy, c = 2x^2 + y^2\], with \(x, y \geq 0\) and \(y\) odd.