Math 445 Homework 4

Due Friday, October 3

13. Show that if $n|m$, and $(10, m) = 1$, then the period of the decimal expansion of $1/n$ divides the period of the decimal expansion of $1/m$.

14. Show that for every $n \geq 2$, $\text{ord}_{3^n}(10) = 3^{n-2}$.
    (Hint: induction! This is not entirely unlike what we did for 7^n....)
    [N.B.: Consequently, the period of the decimal expansion of $1/3^n$ is 3^{n-2} .]

15. Show that if $(3, n) = 1$ (and $(10, n) = 1$), then $\text{ord}_n(10) = \text{ord}_{3n}(10) = \text{ord}_{9n}(10)$.

16. Find the primitive roots of 1 mod 31. (I.e., find all a, $1 \leq a \leq 31$, with $\text{ord}_{31}(a) = 30$.
    (Hint: find one; then use one of our results to quickly find the others.)