Math 445

Take-home Exam (Exam 1)

Due in class on Wednesday, October 29. You are not to discuss the exam, except on trivial matters, with anyone other than the instructor, until after you have (both!) turned in your solutions. The problems are given in approximately the order in which the material was presented in class; this is not necessarily a recommendation to work the problems in that order. Each needed computation should be carried out in full.

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) Find the period of the repeating decimal expansion of 1/53 (by computing the order of the appropriate integer mod the other appropriate integer).

2. (20 pts.) Show that if a and b are both primitive roots of unity modulo the odd prime p, then ab <u>cannot</u> be a primitive root of unity modulo p.

   [Hint: a specific small(er) power of ab will demonstrate this...]

3. (20 pts.) Show that if $n \equiv 1 \pmod 8$, then there are <u>no</u> integers x and y such that $2x^3 - 3y^2 = n$. That is, for $n \equiv 1 \pmod 8$, the equation

   $$2x^3 - 3y^2 = n$$

   has no solutions in the integers.

   [Hint: what values, mod 8, can $3y^2$ take?]

4. (20 pts.) Determine the number of solutions to the power residue equations

   (a) $x^3 \equiv 2 \pmod{13}$

   (b) $x^4 \equiv 2 \pmod{13}$

   (c) $x^5 \equiv 2 \pmod{13}$

5. (20 pts.) Use the RESSOL algorithm to find a solution to the quadratic residue equation

   $$x^2 \equiv 3 \pmod{73} .$$

   [FYI: 5 is a quadratic non-residue mod 73. (It also happens to be a primitive root.)]