36. Let \( h_n/k_n \) (as usual) denote the \( n^{th} \) convergent of the continued fraction expansion of the irrational number \( x \). Show by example that it need not be true that
\[
|x - \frac{a}{b}| < |x - \frac{h_n}{k_n}| \quad \text{implies} \quad b \geq k_{n+1}
\]

37. [NZM, p.344, \# 7.6.3] Show that for any \( c > 2 \), there are only finitely many pairs of integers \( a, b \) with \( |\sqrt{2} - \frac{a}{b}| < \frac{1}{b^c} \).

38. [NZM, p. 333, \# 7.3.6] Let \( p \) be prime and suppose \( u^2 \equiv -1 \pmod{p} \) (so \( p \equiv 1 \pmod{4} \)). Let \([a_0, \ldots, a_n]\) be the continued fraction expansion of \( \frac{u}{p} \), and let \( i \) be the largest integer with \( k_i \leq \sqrt{p} \). Show that \( \left| \frac{h_i}{k_i} - \frac{u}{p} \right| < \frac{1}{k_i \sqrt{p}} \), and \( |h_ip - k_iu| < \sqrt{p} \). Setting \( x = k_i \) and \( y = h_ip - uk_i \), show that \( p|x^2 + y^2 \) and \( x^2 + y^2 < 2p \), so \( x^2 + y^2 = p \).

39. Show that for \( n \) a positive integer that is not a perfect square (translation: the continued fraction expansion of \( \sqrt{n} \) never terminates), that at every stage of the continued fraction expansion of \( x = \sqrt{n} \)
\[
x = [a_0, a_1, \ldots, a_{k-1}, a_k + x_k]
\]
\( x_k \) is always of the form \( x_k = \frac{\sqrt{n} - a}{b} \), where \( a, b \in \mathbb{N} \) and \( b|n - a^2 \). Conclude that the continued fraction expansion of \( \sqrt{n} \) will eventually repeat, with a period of length at most \( n\lfloor \sqrt{n} \rfloor \).

Hint: by induction! In the inductive step, write \( \frac{b}{\sqrt{n} - a} = \frac{\sqrt{n} + a}{c} \), and then find the fractional part of this. For the second half, how long must you wait before the \( x_k \)'s must repeat themselves?