Math 445 Homework 6 (revised)
Due Wednesday, October 27

26. Show that if $p$ is an odd prime and $a$ is a primitive root mod $p$, then \( \left( \frac{a}{p} \right) = -1 \).

27. [Pepin’s Theorem] Show that the Fermat number $F_n = 2^{2^n} + 1$, for $n \geq 1$, is prime if and only if $3^{F_n-1} \equiv -1 \pmod{F_n}$.

28. The primes $p$ for which $x^2 \equiv 13 \pmod{p}$ has solutions consists precisely of those primes lying in certain congruence classes mod 13; which ones?

   [Hint: if you think of the classes as being represented by $-6, \ldots, 0, \ldots, 6$ then you can recycle a lot of your work....]

29. [NZM, p. 148, # 3.3.15] Show that if $p \geq 7$ is an odd prime, then \( \left( \frac{n}{p} \right) = \left( \frac{n+1}{p} \right) \) for at least one of $n = 2, 3, \text{ or } 8$.

   [Hint: it might help to express this in terms of \( \left( \frac{n}{p} \right) \left( \frac{n+1}{p} \right) \)]

30. Compute \( \left( \frac{35}{149} \right) \), \( \left( \frac{39}{145} \right) \), and \( \left( \frac{280}{351} \right) \).