16. [NZM p.59, # 56] Suppose $p$ is prime, and $x^2 \equiv -2 \pmod{p}$. By looking at the numbers $u + xv$ for $u, v$ in some range, show that at least one of the equations

\[ a^2 + 2b^2 = p \text{ or } a^2 + 2b^2 = 2p \]

has a solution.

17. [NZM p.60, # 57] Show that

\[ (a^2 + 2b^2)(c^2 + 2d^2) = (ac - 2bd)^2 + 2(bc + ad)^2 \]

18. [NZM p.60, # 58] Show that if $p$ is prime and odd and $a^2 + 2b^2 = 2p$, then $a$ is even and $b$ is odd. Conclude that $b^2 + 2(\frac{a}{2})^2 = p$ is a solution in the integers.

19. [NZM p.60, # 59] Let $p$ be a prime factor of the number $a^2 + 2b^2$. Show that if $p \nmid a$ or $p \nmid b$ then the equation $x^2 \equiv -2 \pmod{p}$ has a solution.

20. [NZM p.60, # 60] Show that for any prime number $p$, the equation $a^2 + 2b^2 = p$ has a solution $a, b \iff$ the equation $x^2 \equiv -2 \pmod{p}$ has a solution $x$. 