Math 445 Homework 4

Due Wednesday, September 29

16. [NZM p.59, # 56] Suppose p is prime, and $x^2 \equiv -2 \pmod{p}$. By looking at the numbers $u + xv$ for u, v in some range, show that at least one of the equations
$$a^2 + 2b^2 = p \text{ or } a^2 + 2b^2 = 2p$$
has a solution.

17. [NZM p.60, # 57] Show that
$$(a^2 + 2b^2)(c^2 + 2d^2) = (ac - 2bd)^2 + 2(bc + ad)^2$$

18. [NZM p.60, # 58] Show that if p is prime and odd and $a^2 + 2b^2 = 2p$, then a is even and b is odd. Conclude that $b^2 + 2(a/2)^2 = p$ is a solution in the integers.

19. [NZM p.60, # 59] Let p be a prime factor of the number $a^2 + 2b^2$. Show that if $p \nmid a$ or $p \nmid b$ then the equation $x^2 \equiv -2 \pmod{p}$ has a solution.

20. [NZM p.60, # 60] Show that for any prime number p , the equation $a^2 + 2b^2 = p$ has a solution $a, b \Leftrightarrow$ the equation $x^2 \equiv -2 \pmod{p}$ has a solution x.