Math 445 Homework 1
Due Wednesday, Sept. 8

1. (NZM, Problem 1.3.27) Show that if $n$ is not prime, then $n|(n - 1)!$.

2. (NZM, Problem 1.3.32) Show that for $n > 1$, $n^4 + 4$ is never prime.
   (Hint: $f(x) = x^4 + 4$ can be expressed as a product of quadratics; find the factorization!)

3. (NZM, Problem 2.1.27) Show, by induction, that $\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{15}n$ is an integer for every $n \geq 1$. (Note, however, that it is not a multiple of $n$!)

4. Show, by induction on $n$ that
   
   $\left[\text{for every integer } x \geq 1, n! \text{ divides } x(x + 1) \cdots (x + n - 1)\right]$  
   (Hint: prove the statement in brackets $\cdots$ by induction on $x$!)

5. Show that if $a \geq 2$ and $a^n - 1$ is prime, then $n$ must be prime.