Math 445 Homework 8

Due Monday, November 11

33. Find three (different) solutions with $x, y, z > 0$ and $(x, y) = 1$ to the Diophantine equation
$$x^2 + 3y^2 = 7z^2$$

34. Show that the Diophantine equation
$$4x^2 + 11y^3 = 29$$
has no integer solutions.

35. Show that the Diophantine equation
$$57x^2 + 113y^2 = 116z^2$$
has no integer solutions.
(Hint: Use Euler's criterion! The brute force way sounds exhausting...)

36. (NZM, Problem 5.4.4) Show that if $x, y, z \in \mathbb{Z}$ and
$$x^2 + y^2 + z^2 = 2xyz$$
then $x = y = z = 0$.
(One possible approach: think mod 4, and show everyone is even. Divide everyone by two, and show that every number is *still* even, etc. ...)

37. (NZM, Problem 5.4.11) Show that for any prime modulus p, the equation
$$(x^2 - 17)(x^2 - 19)(x^2 - 323) \equiv 0 \pmod{p}$$
always has a solution.
(Hint: Use Euler's criterion! The result is also true for *any* modulus n; one makes use of Hensel's Lemma (p. 87 of NZM) to show it.)