

Math 445 Homework 7

Due Monday, November 4

28. Show that if $k \geq 2$, then the Diophantine equation

$$2x + 3y = k$$

has a solution with $x, y \geq 0$.

29. (NZM, Problem 5.2.12) An integer $N \geq 1$ is called *powerful* if for every prime p , $p|N$ implies $p^2|N$. Show that N is powerful if and only if $N = a^2b^3$ for some integers $a, b \geq 1$.

(Hint: show this for $N = p^k$, p prime, first...)

30. (NZM, Problem 5.3.7) For which values of n does $x^2 - y^2 = n$ have a solution with $x, y \in \mathbb{Z}$?

(Hint: think about $x = y, y + 1, y + 2$ to deal with lots of candidates.)

31. (NZM, Problem 5.3.9) Show that for every integer n , the equation

$$x^2 + z^2 = n + y^2$$

has a solution with $x, y, z \in \mathbb{Z}$

(Hint: Problem # 29 will help.)

32. Show that the Diophantine equation

$$x^2 + y^4 = z^2$$

has infinitely many solutions with $\gcd(x, y) = 1$ and

- (a) y odd,
- (b) y even.