

## Math 445 Homework 6

Due Monday, October 28

25. Let  $h_n/k_n$  (as usual) denote the  $n^{\text{th}}$  convergent of the continued fraction expansion of the irrational number  $x$ . Show by example that it need **not** be true that

$$\left| x - \frac{a}{b} \right| < \left| x - \frac{h_n}{k_n} \right| \text{ implies } b \geq k_{n+1}$$

26. Find two (different!) solutions, with  $x, y \geq 1$ , to the Diophantine equation

$$x^2 - 21y^2 = 1$$

27. For which values of  $N$ ,  $1 \leq N \leq \sqrt{33}$  does

$$x^2 - 33y^2 = N$$

have a solution with  $x, y \in \mathbb{Z}$  ?