Math 445 Homework 6
Due Monday, October 28

25. Let $h_n/k_n$ (as usual) denote the $n^{th}$ convergent of the continued fraction expansion of the irrational number $x$. Show by example that it need not be true that

$$|x - \frac{a}{b}| < \left| x - \frac{h_n}{k_n} \right| \text{ implies } b \geq k_{n+1}$$

26. Find two (different!) solutions, with $x, y \geq 1$, to the Diophantine equation

$$x^2 - 21y^2 = 1$$

27. For which values of $N$, $1 \leq N \leq \sqrt{33}$ does

$$x^2 - 33y^2 = N$$

have a solution with $x, y \in \mathbb{Z}$?