## Math 445 Homework 3

### Due Friday, September 20

For this homework, the symbol "$P(n)$" will denote the length of the period of the repeating decimal expansion of $1/n$ . (I.e., it is the order of 10 mod $n$ .)

10. Show that if $p|q$, then for any $a \geq 1$, $a^p - 1 | a^q - 1$

11. Show than if $(m, n) = 1$, $a^p \equiv 1 \pmod{m}$, and $a^q \equiv 1 \pmod{n}$, then
$$\exp_a\left(\frac{pq}{(p,q)}\right) \equiv 1 \pmod{mn} \quad (\text{where } \exp_a(x) = a^x) \ .$$

12. Show that if $(m, n) = 1$ ( and $(10, m) = (10, n) = 1$ ), then $P(mn) = \dfrac{P(m)P(n)}{(P(m), P(n))}$ .

13. Show that if $(3, n) = 1$ ( and $(10, n) = 1$ ), then $P(3n) = P(n)$ .

14. Show that, for any $n$, $P(n)|P(n^2)$ . Use this to (quickly?) compute $P(49)$ .
    Hint: note that $\phi(49) = 42$, not 48 ....