Math 445 Homework 1

Due Friday, September 6

1. (NZM, Problem 1.2.2) Use the Euclidean algorithm to find the gcd of 1819 and 3587, and express the gcd as $d = 1819n + 3587m$.

2. (NZM, Problem 1.2.6) Show that for any integer n,

   $$6|n(n+1)(n+2) \text{ and } 24|n(n+1)(n+2)(n+3)$$

   Based on this, what more general result might you conjecture to be true?

   Hint: Think about values of n mod 6 and 24, respectively. For the second, it might be less painful to work both mod 3 and mod 8 ....

3. (NZM, Problam 1.3.9) Show that if p is prime and $p \equiv 1 \pmod 3$, then $p \equiv 1 \pmod 6$.

   Hint: What's the alternative?

4. (NZM, Problem 1.3.12) Show that if $x^2 + y^2 = z^2$ for some $x, y, z \in \mathbb{Z}$, then either $3|x$ or $3|y$.

   Hint: Suppose not, and consider what the equation says in $\mathbb{Z}_3$.