Math 417 Problem Set 5

Starred (*) problems are due Friday, February 26.

35. (Gallian, p.96, # 10) If G is a group and $a, b \in G$, show that there is an $x \in G$ with $xax = b$ if and only if there is an element $c \in G$ with $c^2 = ab$.

    [Hint: when you find a way to build such a c from a, b, and x, then that should tell you what x should/could be (in terms of a, b, and c) !]

36. (Gallian, p.119, # 8) Show that the alternating group $A_8 \leq S_8$ contains an element of order 15.

(*) 37. (Gallian, p.119, # 10) Find an element of S_{10} that has the largest order of any element in S_{10}.

38. (Gallian, p.119, # 13) Show that if $\alpha : S \to S$ is a function from a set S to itself and $\alpha(\alpha(x)) = x$ for every $x \in S$, then α must be a <u>bijection</u>.

    [Such a function is usually called an *involution*.]

(*) 39. Show that if $\alpha \in S_n$ has $|\alpha|$ <u>odd</u>, then α is an <u>even</u> permutation!

40. (Gallian, p.120, #32) If $\beta = (1, 2, 3)(1, 4, 5)$, express β^{99} as a product of disjoint cycles.

41. (Gallian, p.121, #48) Show that in the symmetric group S_7, there is <u>no</u> element $x \in S_7$ so that $x^2 = (1, 2, 3, 4)$. On the other hand, find two distinct elements $y \in S_7$ so that $y^3 = (1, 2, 3, 4)$.

(*) 42. (Gallian, p.122, # 69) Show that every element of S_n can be written as a product of transpositions of the form $(1, k)$ for $2 \leq k \leq n$. (Assume that $n > 1$ so that you don't have to worry about the philosophical challenges of $S_1 = \{()\}$...)

    [Hint: why is it enough to show that this is true for transpositions?]