Math 417 Problem Set 3

Starred (*) problems are due Friday, February 12.

(*) 18. (Gallian, p.70, #32) Show that if G is a group and $H, K \subseteq G$ are subgroups of G, then their intersection $H \cap K$ is also a subgroup of G. Does this extend to the intersection of any number of subgroups of G ?

19. (Gallian, p.70, #16) If G is a group, and $H \subseteq G$ is a subset of G so that, whenever $a, b \in H$ we have $a^{-1}b^{-1} \in H$, is this enough to guarantee that H is a subgroup of G? If yes, explain why! If not, give an example which shows that it doesn't work.

    [Hint: if $a \in H$, start listing other elements that you can guarantee are in H ...]

20. (Gallian, p.69, #15 [and then some...]) Show that if G is a group with $g \in G$ and $n = |g| < \infty$, and k is relatively prime to n, then there is an $h \in G$ with $g = h^k$.

    [Hint: This should be true even if we replace G with a subgroup of G which contains g, e.g., $H = \langle g \rangle$!]

21. (Gallian, p.57, #38) Show that if G is a group and $a, b \in G$, then there is an $x \in G$ so that $axb = bxa$. Show, therefore, that if G has the property that whenever $axb = cxd$ we must have $ab = cd$ ('middle cancellation'), then G must be abelian.

(*) 22. (Gallian, p.72, #46) Suppose that G is a group and $g \in G$ has $|g| = 5$. Show that the centralizer of g, $C(g) = C_G(g) = \{x \in G \ : \ xg = gx\}$, is equal to the centralizer of g^3, $C_G(g^3)$.

    [Hint: show that anything that commutes with g must commute with g^3, and vice versa! What, if anything, is special about the numbers 5 and 3 in this problem?]

23. Show, by example, that if G is a group and $g, h \in G$ have $|g| = |h| = 2$, that $|gh|$ can be any natural number (including ∞ !).

    [Hint: Problem #1 !]

24. Show, by example, that if G is a group and $g, h \in G$ have $|g| = 2$ and $|h| = 3$, that $|gh|$ can be any natural number (including ∞ !) except 1.

(*) 25. (Gallian, p.74, #68) Let $G = GL_2(\mathbb{R})$ = the 2×2 invertible matrices, under matrix multiplication, and let $H = \{A \in GL_2(\mathbb{R}) \ : \ \det(A) = 2^k \text{ for some } k \in \mathbb{Z}\}$. Show that H is a subgroup of G.

26. If G is an abelian group and $n \in \mathbb{Z}$, show that $H_n = \{g \in G \ : \ g = x^n \text{ for some } x \in G\}$ is a subgroup of G. Give an example where this fails if G is not abelian.