Math 325 Problem Set 5

Due Friday, February 24

16. Show that if $a, b \in \mathbb{R}$ and $0 < a \le b$, then $\sqrt{a} \le \sqrt{b}$. [Suppose not.....]

17. [Lay, p.173, # 17.15] Show that as $n \to \infty$ we have

    (a): $\sqrt{n+1} - \sqrt{n} \to 0$.

    (b): $\sqrt{n^2+1} - n \to 0$.

    (c): $\sqrt{n^2+n} - n \to \dfrac{1}{2}$.

18. [Lay, p.180, # 18.7] Define the sequence $(a_n)_{n=1}^{\infty}$ by $a_1 = \sqrt{6}$, and, for $n > 1$,

$$a_n = \sqrt{6 + a_{n-1}}$$

(so, e.g, $a_2 = \sqrt{6+\sqrt{6}}$, $a_3 = \sqrt{6+\sqrt{6+\sqrt{6}}}$, etc.). Show that the sequence is monotone and bounded, and determine what it converges to.

[Hints: show monotone by induction! (after figuring out monotone *what*...). For a bound, what value would make your inequality an equality? For the limit, note that $a_n \to L$ implies that $a_{n+1} \to L$.]

19. [Lay, p.180, # 18.5 (sort of)] Show by example that, if $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ are both monotone sequences, then the following conclusions need <u>not</u> be true:

    (a): $(c_n)_{n=1}^{\infty}$, where $c_n = a_n + b_n$, is monotone.

    (b): $(d_n)_{n=1}^{\infty}$, where $d_n = a_n b_n$, is monotone.

    What additional hypotheses (if any), will make one or both of these conclusions <u>true</u>?